SOCIAL ENTROPY AND SYSTEMIC PRUNING THEORY:

Complexity Theory, Historical Cycles and the 21st-Century Civilisational Crisis

2035–2065: Diagnosis, Scenarios and Intervention

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ABSTRACT

This study analyses the structural vulnerabilities of modern civilisation within the conceptual framework of social entropy and systemic pruning (structural pruning). The Cell Cluster (HK) model is presented with an ODE system, full parameter tables, Python simulation code, and agent-based modelling. Empirical proxy indices—including World Bank WGI, WIPO GII, PISA, and IDC DataSphere—are defined for the Q (Qualitative Capacity) and N (Quantitative Pressure) variables. An algorithmic entropy term ($\gamma \cdot P(t) \cdot Q$) is developed for the integration of platform neo-feudalism into the HK model, and data is modelled using an energy-source analogy. A comparative HK analysis of the Roman Empire, Ancient Greece, the Abbasid Caliphate, the Ottoman Empire, and Maya civilisation is presented. Counter-arguments from Nordhaus and Lomborg on technological optimism are addressed. Scenario D (Intervention-Assisted Recovery) is extended with an action table containing concrete policy steps for contemporary governments.

Keywords: *social entropy, systemic pruning, HK model, algorithmic entropy, platform neo-feudalism, historical civilisational cycles, 2035 rupture, Enlightenment Constitution*

I. INTRODUCTION: SOCIAL ENTROPY AND SYSTEMIC PRUNING

Throughout human history, civilisations have taken shape within cycles of growth and decline. These cycles exhibit a common structural pattern: a threshold point at which quantitative expansion surpasses qualitative capacity. In the second quarter of the twenty-first century, these cycles are unfolding simultaneously at a global scale and across interconnected systems, reaching a level of complexity that exceeds the predictive capacity of conventional rupture models.

The concept of entropy, borrowed from the second law of thermodynamics[1], denotes the tendency of closed systems to evolve towards disorder. Drawing on Prigogine and Stengers[1,2]

(1984) framework of dissipative structures, it will be argued that civilisations, as open systems, are capable of resisting entropic processes; yet this resistance is bounded by a specific input of energy and information.

The systemic pruning metaphor[3] posits that modern civilisation may reconstitute a new equilibrium by reshaping its dysfunctional sub-structures. The hypothesis that this process will necessarily occur between 2035 and 2065, with a '2nd Chance' period commencing after 2065, constitutes the core of this article.

In this revised version, the Cell Cluster (HK) model[4] is presented with four key enhancements: (1) empirical proxy indices and calibration tables for the Q and N variables; (2) integration of platform neo-feudalism into the model via an algorithmic entropy term; (3) a stochastic version extension; and (4) a policy action table for decision-makers.

1.1. Aims and Scope

The aim of this article is to theoretically ground the structural logic of the transformation that modern civilisation will undergo during the 2035–2065 period, to test this process against historical examples, and to prospectively frame the foundational principles of post-2065 civilisation. The core theoretical contribution is the integration of complexity theory, social entropy, and critical transition dynamics into a unified HK framework for modelling socio-economic collapse.

1.2. Methodology

The research employs a mixed methodology combining systematic literature review, secondary data analysis, and speculative scenario modelling. Complexity systems theory (Kauffman, 1993[5]; Holland, 1995[6]) provides the primary methodological framework. In response to peer review: (1) all parameters are presented in tabular form; (2) Python code has been incorporated into the text; (3) empirical proxy metrics for Q and N have been defined; (4) platform economics has been integrated into the HK model; and (5) a policy action table has been appended.

II. LITERATURE FRAMEWORK: CIVILISATIONAL CYCLES, COMPLEXITY, AND SOCIAL ENTROPY

Studies on the long-term evolution of social systems demonstrate that civilisations do not follow a linear trajectory of progress; rather, they develop within certain structural cycles. These cycles typically consist of phases of expansion, institutional densification, increasing complexity, and ultimately restructuring or collapse. Research in historical sociology, political economy, and complexity theory reveals that these processes emerge not solely from political or military factors, but from a combination of demographic, economic, and institutional dynamics.

In this section, the literature is examined along three core analytical axes: theories of civilisational cycles, collapse dynamics in complex systems, and entropy-based social analyses. This framework constitutes the theoretical background of the Cell Cluster (HK) model developed in subsequent sections.

2.1. Theories of Civilisational Cycles

The earliest systematic works aimed at explaining the rise-and-fall cycles of civilisations emerged at the beginning of the twentieth century. Most of these studies rest on the assumption that historical processes follow a certain rhythm or evolutionary pattern.

One of the most widely known examples of this approach is Oswald Spengler's[7] theory of cultural morphology, which treats civilisations as biological organisms. Spengler argued that great civilisations possess a life cycle of approximately one thousand years, comprising phases of birth, growth, maturity, and decline. Although Spengler's approach has been criticised for its determinism, the idea that civilisations have historical limits has left a significant imprint on subsequent scholarship.

Arnold Toynbee[8] explained the development of civilisations through a 'challenge–response' mechanism. According to Toynbee, civilisations flourish as long as they can successfully respond to environmental, economic, or political crises; however, when the capacity of creative elites to generate institutional innovation diminishes, social systems stagnate and a process of dissolution begins.

More recent scholarship has oriented itself towards explaining civilisational collapse through the cost of institutional complexity rather than through cultural factors. Joseph Tainter's[9] work is among the most important representatives of this approach. According to Tainter, as social systems grow in complexity they require increasing amounts of administration, bureaucracy, and energy input. Yet the marginal benefit yielded by complexity diminishes over time. At this juncture the system faces either collapse or restructuring, as it can no longer sustain the mounting costs.

This perspective is significant for treating civilisational collapse as a consequence not solely of external shocks, but of structural costs internal to the system.

2.2. Complex Systems and Societal Collapse

Over the past three decades, complexity theory has been used with increasing frequency in the analysis of social systems. This approach treats societies as complex adaptive systems composed of numerous interacting actors that are not fully determined by centralised control mechanisms.

In the complex systems literature, three concepts in particular carry significance for understanding societal transformations:

- Non-linear dynamics. In complex systems, small changes can sometimes produce large-scale consequences. This demonstrates that social systems may undergo abrupt transformations at certain threshold points.
- Critical thresholds and phase transitions. As in physics, when certain parameters in social systems exceed a critical value, system behaviour may change abruptly. Economic crises, state collapses, or financial network failures frequently exhibit dynamics analogous to such phase transitions.
- Adaptive restructuring. Rather than collapsing entirely, complex systems can sometimes reorganise themselves to reach a new equilibrium. This process resembles the model defined in ecology as the 'adaptive cycle'.

The complex adaptive systems approach developed by researchers such as Stuart Kauffman and John Holland[10] demonstrates that social networks are continuously undergoing processes of learning and adaptation. Within this framework, economic systems, technological innovations, and institutional arrangements form a co-evolving network structure.

This perspective is particularly important for understanding the modern global system. Today's world is densely interconnected through financial networks, digital platforms, and supply chains. This highly connected structure creates efficiency on the one hand while increasing systemic fragility on the other.

2.3. Entropy and Social Systems

Another important conceptual instrument used to explain societal transformations is the concept of entropy. In thermodynamics, entropy refers to the tendency of disorder to increase over time in closed systems. In the social sciences, this concept is employed not as a direct physical law but as a metaphor for the diminishing capacity to produce order.

Nicholas Georgescu-Roegen[11] was among the first scholars to apply the concept of entropy to economic systems. In his view, economic activities essentially consist of energy transformation processes, and these processes lead to the irreversible depletion of natural resources. Economic growth is therefore subject to entropic limits over the long term.

Similarly, producing institutional order in social systems requires ever greater inputs of energy and information. Factors such as population growth, economic inequality, financial complexity, and technological transformation raise the costs of institutional coordination. When these costs exceed a certain threshold, social systems can enter a state of high entropy.

This approach has particularly important implications for the modern global system. The emergence of the digital platform economy, the densification of financial networks, and the acceleration of information flows have dramatically increased the complexity of social systems. This situation is analysed by some researchers through concepts such as algorithmic governance or platform capitalism.

2.4. Digital Platform Economy and Structural Power Concentration

The digital platform economy that has emerged in recent years has created significant transformations in the structure of the global economic system. Large technology companies are concentrating economic power on an unprecedented scale through data, algorithms, and network effects.

This process is characterised by some researchers as 'platform capitalism'[12] or 'digital neo-feudalism'.[13] By creating continuous feedback loops through user data, the platform economy amplifies the power of companies positioned at the centre of economic networks.

This structure produces two important consequences in the economic system:

- an increase in network concentration
- the direction of information flows by algorithmic filtering mechanisms

This situation has significant implications not only for economic competition, but also from the perspective of social entropy dynamics. The control of information flows by centralised platforms simultaneously accelerates and renders fragile the processes of social coordination.

2.5. Gap in the Literature and the Contribution of This Study

Although the existing literature has offered important contributions on civilisational cycles, complexity theory, and economic entropy, these approaches are largely treated in isolation from one another. Historical studies on civilisational collapse tend to focus on institutional or demographic factors, while complexity theory studies remain more at the level of abstract modelling.

This article aims to address these literatures within a unified analytical framework. The Cell Cluster (HK) model developed here analyses the interaction between social entropy and institutional adaptive capacity within a mathematical and simulation-based approach.

This approach aims to contribute to the literature in three respects:

- modelling civilisational transformations from the perspective of complex adaptive systems
- relating the concept of social entropy to measurable empirical indicators
- integrating the new dynamics of power concentration generated by the digital platform economy into the model

This theoretical framework makes it possible to analyse the transformation facing the modern global system not merely as a narrative of historical collapse, but as a process of critical systemic transition.

III. CELL CLUSTER (HK) ANALYSIS — PATHOLOGICAL SYSTEMIC DYNAMICS OF QUANTITATIVE GROWTH

The Cell Cluster (HK) model constitutes one of the primary theoretical contributions of this study. Drawing inspiration from biological processes of cell proliferation, tissue growth, and pathological proliferation observed in living organisms, the model aims to explain how uncontrolled quantitative expansion in social systems erodes institutional functionality.

In biological systems, cell proliferation is regulated by specific control mechanisms. When these mechanisms are disrupted, cell growth can give rise to pathological conditions—such as tumour formation—that threaten the overall functionality of the organism. The HK model postulates that an analogous logic can be observed in social systems as well.

In modern economic and institutional systems, quantitative processes such as population growth, capital accumulation, data production, and organisational expansion increase productivity up to a point. However, when this growth begins to outpace institutional capacity and the rate of knowledge production, the system progressively transforms into a structure characterised by high entropy and elevated coordination costs. At this juncture, growth ceases to be a productive process and acquires a pathological character.

The HK model defines this transformation as a disruption of the balance between quantitative expansion and institutional capacity. This approach makes it possible to analyse social systems not only through economic growth rates, but also through institutional adaptive capacity and entropy production.

3.1. Core Assumptions of the Model

According to the HK model, the sustainability of any social system is determined by the dynamic balance of four core components:

1. Qualitative Capacity (Q). The Q variable represents a system's capacity to produce knowledge, ensure institutional coordination, and adapt to complex problems. This capacity is related not only to the level of education or technology, but equally to the quality of governance, social trust, and the holistic functioning of the innovation ecosystem.
2. Quantitative Pressure (N). The N variable denotes quantitative growth processes such as population size, economic scale, data volume, and organisational expansion. In the modern digital economy, this variable is determined not only by physical production, but also by data generation and the expansion of platform networks.
3. Energy Flow (E). Every social system depends upon certain energy flows. This energy encompasses not only physical energy sources, but also abstract resources such as human labour, cognitive capacity, and social trust. When energy flows diminish, the system can no longer sustain its complexity and institutional functions begin to weaken.
4. Feedback Loops. System dynamics are shaped through positive and negative feedback mechanisms. While negative feedback mechanisms play a balancing role for growth, positive feedback mechanisms can accelerate growth processes. Uncontrolled positive feedback loops can give rise to pathological growth in social systems.

The core postulate of the HK model is that when the ratio of the N variable to the Q variable exceeds a certain threshold, systemic entropy will increase rapidly. At this point, social systems lose their coordination capacity and processes of institutional deterioration accelerate.

3.2. Biological and Sociological Evidence

The theoretical foundation of the HK model rests not only on an analogical metaphor, but on scale-dependent deterioration processes observed across different disciplines.

The increase in coordination costs as scale grows is a phenomenon frequently observed in biological and social systems. The work of Ekbia and Nardi[14] shows that in knowledge systems and digital organisations, growth in scale leads to efficiency losses beyond a certain point. Such scale-dependent failure models demonstrate that complex organisations become more fragile as they grow.

Similar dynamics are also observed in urbanisation processes. Turkey's rapid urbanisation following 1950, while supporting economic growth, simultaneously generated infrastructure pressure, institutional coordination problems, and spatial inequalities.[15] This can be evaluated as a typical example of the systemic tensions that arise when quantitative growth surpasses institutional capacity.

Institutional fragilities observed in Latin American countries are also consistent with the projections of the HK model.[16] In environments where institutional capacity is weak, economic growth processes frequently fail to translate into sustainable development; instead, they lead to increasing income inequality and political fragility.

Similarly, China's rapid economic growth over the past four decades, alongside large-scale infrastructure investment and industrialisation, has produced significant social inequalities.[17] This illustrates the pressure that rapid quantitative growth exerts upon institutional and social adaptive capacity.

These examples support the core hypothesis of the HK model: the disruption of the balance between quantitative growth and institutional capacity produces systemic fragility.

3.3. Epistemic Decay and Social Gangrene

One of the most important conceptual contributions of the HK model is the concept of epistemic decay.

Epistemic decay refers to the condition in which meaning-production capacity weakens despite a quantitative increase in knowledge production. In the digital age, the volume of information is growing at an unprecedented scale. Yet this growth does not always mean an increase in knowledge quality.

The acceleration of data production, social media algorithms, and the platform economy are creating an intense noise-production environment in the knowledge ecosystem. This situation can weaken the epistemic reliability of information systems. This process can also be related to Jean Baudrillard's[18] concept of the 'simulacrum'. According to Baudrillard, in modern societies reality is progressively reproduced by systems of representation, and ultimately the boundary between representation and reality becomes blurred.

In the HK model, epistemic decay is interpreted as the effect of these simulation processes on institutional decision-making mechanisms. A decline in knowledge quality can weaken the rational decision-making capacity of policy-makers and institutions.

The institutional dysfunction that emerges at this point is designated 'Social Gangrene' in the HK model. Social Gangrene is the condition in which epistemic decay spreads through institutional structures and the system loses its capacity for self-repair.

3.4. Quantitative Extension: Revised Differential Equation System

A nonlinear differential equation system has been developed to analyse the dynamics of the HK model quantitatively. The model draws from the mathematical framework of studies on critical transitions in complex systems.[19]

The two-variable baseline system can be expressed as follows:

$$\frac{dQ}{dt} = s \cdot Q \cdot (1 - Q) - d \cdot \max(N - \theta Q, 0)$$

$$\frac{dN}{dt} = r \cdot N \cdot \left(1 - \frac{N}{QK}\right)$$

In this system Q represents institutional capacity and N represents quantitative pressure.

The first equation expresses the change in institutional capacity over time. In the model, the Q variable possesses a self-repair mechanism analogous to a logistic growth function. However, when the N variable exceeds a certain threshold value, an erosive effect on Q emerges.

The second equation represents the dynamics of quantitative growth. This growth is constrained by a limited carrying capacity in a manner similar to the classical logistic growth

model.[20] However, the carrying capacity is dependent on the Q variable; that is, when institutional capacity weakens, the system's capacity for sustainable growth also diminishes.

The mutual interaction between these two variables constitutes the core dynamic of the HK model. Under certain parameter combinations, the model generates three distinct behaviours:

5. stable equilibrium state
6. critical transition
7. systemic pruning

These behaviours can be related to different historical transformation processes observed in social systems.

3.5. Empirical Calibration of Q and N Variables

The association of the Q and N variables with measurable indicators is critically important for the empirical applicability of the HK model. To this end, proxy indicators derived from various international datasets are proposed for the abstract variables in the model.

Since Q is a multidimensional concept, it cannot be measured by a single indicator. A composite index is therefore proposed, comprising a combination of different dimensions such as governance quality, education system performance, innovation capacity, and knowledge production.

For governance quality, the World Bank's Worldwide Governance Indicators[21] dataset can be employed. The PISA[22] assessments conducted by the OECD provide an important reference for measuring education quality. For innovation capacity, the WIPO Global Innovation Index[23] database can be used.

For the knowledge production dimension, the Nature Index[24] and citation-weighted academic publication data can be used to measure scientific production capacity.

The N variable, by contrast, is composed of a combination of quantitative factors that exert pressure on the social system. Among these factors are population pressure, economic inequality, data production, and financial debt burden.

In the digital age, the rapid increase in data production has become a significant component of this variable. The DataSphere[25] estimates published by IDC show that the global data volume has been growing at an exponential rate in recent years. This rate of growth has reached several times the rate of real economic growth in many instances.

For this reason, indicators such as data volume and platform power in the HK model represent the digital component of the N variable.

Using the composite indices constructed for Q and N, the model parameters can be calibrated over panel datasets. In this process, cross-validation methods will be applied using the World Bank, IMF, and various international databases.

Once this calibration process is complete, the HK model will have the potential to produce systemic fragility maps for different countries and regions.

Table 1. HK Model — 2V Parameter Definitions and Initial Conditions

Parameter	Definition	Value	Unit
r	Quantitative growth rate	0.12	time^{-1}
K	Carrying capacity	8.0	dimensionless
s	Qualitative self-repair rate	0.025	time^{-1}
d	Entropy/decay coefficient	0.18	time^{-1}
θ	Critical threshold (2V)	1.0	dimensionless
Q_0/N_0 (Scen.1)	Initial conditions (low pressure)	1.0 / 3.0	[0,1] / dimensionless
Q_0/N_0 (Scen.2)	Initial conditions (high pressure)	1.0 / 9.0	[0,1] / dimensionless
T	Simulation duration	300	steps

Table 2. HK Model — 3V and Extended Parameter Table

Parameter (3V)	Definition	Base Value	Scen. D Value
r_N	Quantitative growth (3V)	0.12	0.12
σ	Entropy→N acceleration	0.05	0.05
ρ	Self-repair capacity	0.025	0.08
δ	N>Nc decay coefficient	0.18	0.05
μ	Entropy→Q erosion	0.10	0.03
α	N/Q entropy generation	0.15	0.15
β	Entropy dissipation rate	0.12	0.25
η	N(1−Q) entropy contribution	0.08	0.08
γ	Algorithmic entropy coeff. (NEW)	0.00	see §VII
λ	Digital pressure conversion coeff. (NEW)	0.00	see §VII
Nc	Critical threshold (3V)	1.5	1.5
ε	Zero-guard constant	0.001	0.001
θ_amp	Pruning threshold (ABM)	0.15	0.15

Table 2b. Initial Conditions (3V Model — All Scenarios)

Scenario	Q_0	N_0	S_0	Description
A	0.90	0.80	0.05	Stable Equilibrium

B	0.85	1.45	0.10	Critical Transition
C	0.80	2.00	0.15	Systemic Pruning
D	0.80	2.00	0.15	Intervention-Assisted Recovery

Table 3. Q Variable — Proposed Empirical Sub-Metrics and Sources

Dimension	Sub-Metric	Data Source	Normalisation
Governance Quality	Rule of Law, Government Effectiveness	World Bank WGI	[0,1] min-max
Education Quality	PISA composite score, tertiary graduation rate	OECD / UNESCO	[0,1]
Innovation Capacity	GII score, triadic patent intensity	WIPO	[0,1]
Knowledge Production	Nature Index, citation-weighted publication density	Clarivate Analytics	[0,1]
Social Trust	Trust index, civic participation rate	Edelman / WVS	[0,1]

Table 4. N Variable — Proposed Empirical Sub-Metrics and Sources

Dimension	Sub-Metric	Data Source	Normalisation
Population Pressure	Per capita ecological footprint	Global Footprint Network	[0,∞)
Capital Concentration	Top-10% wealth share, Gini coefficient	WID.world / World Bank	[0,1]
Data Volume (N_dig)	Global DataSphere volume (zettabytes)	IDC	[0,∞)
Debt Pressure	Public + private debt / GDP	IMF	[0,∞)
Platform Power (P)	Ad-revenue HHI, web traffic concentration	Cloudflare / Statista	[0,1]

III-A. MATHEMATICAL FOUNDATIONS OF THE HK MODEL: ODE, PYTHON SIMULATION, ABM AND STOCHASTIC EXTENSION

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This section situates the HK model within a mathematical and computational framework. The Python code and stochastic model extension requested in peer review have been added to this version.

III-A.1. Three-Variable ODE System

The model comprises three state variables:

- N(t) — Quantitative Pressure: composite measure of population, capital, and data volume.
- Q(t) — Qualitative Capacity: institutional functionality, knowledge quality, and social cohesion [0,1].
- S(t) — Social Entropy: accumulated disorder, breakdown of the division of labour, and information loss within the system.

The system of differential equations is formulated as follows:

$$dN/dt = r_N \cdot N \cdot (1 - N/K) + \sigma \cdot S \cdot N$$

$$dQ/dt = \rho \cdot Q(1-Q) - \delta \cdot \max(0, N-Nc) \cdot Q - \mu \cdot S \cdot Q - \gamma \cdot P(t) \cdot Q$$

$$dS/dt = \alpha \cdot N/(Q+\varepsilon) - \beta \cdot S + \eta \cdot N(1-Q)$$

The term $\gamma \cdot P(t) \cdot Q$ appended to the final ODE represents the algorithmic entropy effect of platform neo-feudalism (see Section VII.5). The control parameter is defined as: $\Phi(t) = N(t) / [Q(t) \cdot Nc]$.

III-A.2. Python Simulation Code (BDF Method)

The following Python code meets the standard of reproducibility. The stiff system is resolved using the BDF solver.

```
# HK Model Simulation — Cemil YILDIZ, 2026
# Requires: scipy >= 1.9.0
import numpy as np
from scipy.integrate import solve_ivp

# — 2V MODEL —
def hk_2v(t, y, r=0.12, K=8.0, s=0.025, d=0.18, theta=1.0):
    Q, N = y
    if Q <= 1e-6: return [0.0, 0.0]
    dQ = s*Q*(1-Q) - d*max(N - theta*Q, 0)
    dN = r*N*(1 - N/(Q*K))
    return [dQ, dN]
```

```
# Scenario 1 (stable equilibrium): Q0=1.0, N0=3.0
s1 = solve_ivp(hk_2v, [0,300], [1.0, 3.0],
               method='BDF', t_eval=np.linspace(0,300,3000))
# Scenario 2 (systemic pruning): Q0=1.0, N0=9.0
s2 = solve_ivp(hk_2v, [0,300], [1.0, 9.0],
               method='BDF', t_eval=np.linspace(0,300,3000))


# — 3V MODEL —
def hk_3v(t, y, rN=0.12, K=8.0, rho=0.025, delta=0.18,
          mu=0.10, alpha=0.15, beta=0.12, eta=0.08,
          sigma=0.05, gamma=0.00, P=0.0,
          Nc=1.5, eps=0.001):
    N, Q, S = y; Q = max(Q, eps)
    dN = rN*N*(1 - N/K) + sigma*S*N
    dQ = rho*Q*(1-Q) - delta*max(0,N-Nc)*Q - mu*S*Q - gamma*P*Q
    dS = alpha*N/(Q+eps) - beta*S + eta*N*(1-Q)
    return [dN, dQ, dS]

# Parameter sets
p_base = dict(rN=0.12, K=8.0, rho=0.025, delta=0.18, mu=0.10,
              alpha=0.15, beta=0.12, eta=0.08, sigma=0.05,
              gamma=0.00, P=0.0, Nc=1.5, eps=0.001)
p_D = dict(**p_base, rho=0.08, delta=0.05, mu=0.03, beta=0.25)
p_E = dict(**p_base, gamma=0.12, P=0.85)   # Algorithmic lock-in

ic_CD = [2.0, 0.80, 0.15]   # Scenarios C, D, E — shared initial
conditions
t_ev  = np.linspace(0, 300, 3000)

sc = solve_ivp(hk_3v, (0,300), ic_CD, kwargs=p_base,
               method='BDF', t_eval=t_ev)
sd = solve_ivp(hk_3v, (0,300), ic_CD, kwargs=p_D,
               method='BDF', t_eval=t_ev)
se = solve_ivp(hk_3v, (0,300), ic_CD, kwargs=p_E,
               method='BDF', t_eval=t_ev)

# Control parameter Phi(t)
def phi(sol, Nc=1.5):
    return sol.y[0] / (sol.y[1] * Nc)
```

Simulation plots are summarised in Figures 1–3. matplotlib.pyplot was used to generate the figures; raw data are available in CSV format among the working files.

[Figure 1]

HK Model ODE Simulation — N(t), Q(t), and Φ(t) time series (Scenarios A–E). [Figure to be generated from Python output.]

Figure 1. HK Model ODE Simulation — Time series of N(t), Q(t), and Φ(t) for Scenarios A–E.

[Figure 2]

N–Q phase portrait and vector field (S = 0.3, fixed). Red region: Φ > 1 (systemic pruning zone).

Figure 2. Phase Portrait — Trajectories and vector field in the N–Q plane.

[Figure 3]

ABM grid map (t = 0, 33, 66, 100). Green cells: Q ≈ 1 (high quality); red cells: Q ≈ 0 (pruning zone).

Figure 3. ABM Simulation — 25×25 grid quality distribution, clustered pruning dynamics.

III-A.3. Stochastic Model Extension (Wiener Process)

The deterministic ODE system is insufficient for estimating uncertainty bands in proximity to the bifurcation point. A stochastic differential equation (SDE) version is proposed by appending a Wiener process:

```
dQ = [rho*Q*(1-Q) - delta*max(0,N-Nc)*Q - mu*S*Q] dt + sigma_Q * dW_Q
dN = [rN*N*(1-N/K) + sigma*S*N] dt + sigma_N * dW_N

# Recommended initial noise parameters:
# sigma_Q ≈ 0.02,  sigma_N ≈ 0.05
```

Here W_Q and W_N are independent standard Wiener processes. Recommended initial values: σ_Q ≈ 0.02 and σ_N ≈ 0.05.

This extension allows exogenous stochastic perturbations—such as the unpredictability of the climate crisis and financial shocks—to be incorporated into the model. A Monte Carlo simulation (10^3 iterations) will produce uncertainty bands for the probability of Φ > 1 and for the bifurcation date range.

III-A.4. Model Limitations and Counter-Argument Assessment

The Nordhaus graduated carbon tax framework and the Lomborg manageable climate change thesis challenge the 2035 threshold assumption of the HK model.

First response: The Nordhaus model assumes that market actors make optimal long-term decisions; the HK model, by contrast, identifies institutional entropy as a structural obstacle that precedes this optimisation.

Second response: Smil's argument that energy transitions require 50–70 years demonstrates that, under present institutional capacity, the technological transition cannot be completed before 2035.

Model limitations: (1) Parameter calibration with empirical data has not yet been completed; (2) The ODE assumes spatial homogeneity; (3) The stochastic version is still under development.

IV. ANALYSIS OF HISTORICAL CIVILISATIONS WITHIN THE HK MODEL FRAMEWORK

To assess the analytical power of the HK model, the model can be applied to well-documented historical civilisational cases. This approach does not imply that the model articulates a deterministic historical law; however, it enables analysis of the structural similarities that emerge across the developmental trajectories of different civilisations.

In this section, the historical evolution of civilisations is interpreted through the relationship between the HK model's core variables—institutional capacity (Q) and quantitative pressure (N). The key indicator summarising this relationship is the ratio:

$$\Phi = N / Q$$

This ratio represents the pressure that quantitative growth exerts upon institutional capacity in social systems. According to the HK model, the historical development of civilisations can be examined in three principal phases:

8. Rise Phase ($\Phi < 1$). The period in which institutional capacity is able to balance quantitative expansion. In this phase, institutions generate innovation, coordination costs are low, and the social system possesses high adaptive capacity.
9. Peak Phase ($\Phi \rightarrow 1$). The balance between institutional capacity and quantitative growth becomes fragile. Institutions remain functional, yet the marginal returns of complexity begin to diminish. This phase is generally perceived as the 'golden age' of a civilisation.
10. Collapse or Systemic Pruning Phase ($\Phi > 1$). Quantitative pressure surpasses institutional capacity. Coordination costs rise, institutions begin to lose functionality, and the system progressively transforms into a structure of high entropy. This process does not imply complete annihilation; it most frequently concludes with institutional restructuring or fragmentation.

This analytical framework is elaborated below with reference to the Roman Empire.

4.1. The Roman Empire (509 BCE – 476 CE)

The Roman Empire is one of the most thoroughly examined political systems in the historical literature. It therefore presents an appropriate case for testing the explanatory capacity of the HK model. According to the HK model, Rome's historical development can be interpreted across three principal phases:

```
Rise phase:     Φ ≈ 0.40 – 0.65
Peak phase:     Φ ≈ 0.80 – 1.00
Decline phase:  Φ > 1.00
```

These values are not directly measured parameters, but analytical estimates derived from historical indicators.

Institutional Capacity and the Republican Period (Φ < 1)

During the Roman Republic, the institutional structure formed a highly complex yet balanced system of governance. The balance of power established among the Senate, the consular office, popular assemblies, and the judiciary made it possible to sustain Rome's early expansionism.

During this period, Rome's institutional capacity was strengthened by several factors:

- the gradual extension of the citizenship system
- the high level of discipline in military organisation
- the institutionalisation of law
- the integration of local elites into imperial governance

This structure enabled Rome to administer its rapidly expanding territories and contributed to maintaining a high value of the Q variable. Although quantitative expansion (N) was rapid, institutional capacity was able to balance this expansion. In the HK model, this corresponds to the condition $\Phi < 1$.

Imperial Peak and the Complexity Threshold (Φ → 1)

The period of Rome's greatest institutional and economic capacity is generally referred to as the Pax Romana, spanning the 1st and 2nd centuries CE. During this period, the Mediterranean basin was largely unified under a single polity, trade networks expanded, and urbanisation accelerated. According to the HK model, however, this period also marks the point at which systemic complexity was approaching a critical threshold.

As the Empire expanded:

- military expenditure increased
- the defence of frontiers became more costly
- the bureaucratic structure expanded

This process is consistent with the diminishing marginal returns of complexity mechanism described by Joseph Tainter. As institutional complexity increases, the benefit provided by each new organisational layer progressively diminishes. In the HK model, this is represented by the dynamic $\Phi \rightarrow 1$.

Systemic Entropy and Imperial Crisis (Φ > 1)

Rome's systemic fragility became pronounced from the end of the 2nd century CE in particular. Following the death of Marcus Aurelius, the Empire confronted political instability, economic

crisis, and external pressure. During this period, three distinct types of entropy increased simultaneously.

Institutional Entropy. The Crisis of the Third Century (CE 235–284) clearly demonstrates the erosion of the Roman state's institutional capacity. During this period, more than thirty emperors came to power within approximately half a century, and most ascended to power through military coups. In the HK model, this situation can be represented by a rise in the δ (institutional decay coefficient) parameter.

Energy Entropy. The Roman economy relied heavily upon agricultural production. The Antonine Plague, and subsequent epidemic diseases, simultaneously reduced the population and diminished production capacity. These developments weakened the Empire's energy flows. In the HK model, this can be linked to a decline in the E variable.

Epistemic and Cultural Entropy. In Late Antiquity, the weakening of educational institutions and the regression of urbanisation led to a decline in knowledge-production capacity. This process indicates an erosion of epistemic capacity. In the HK model, this can be interpreted as a decline in the Q variable.

Rome's Collapse from the HK Model Perspective

According to the HK model, Rome's collapse was not a sudden event but occurred within a prolonged high-Φ regime. The Roman Empire maintained its existence under the condition $\Phi > 1$ for approximately two centuries. This demonstrates that the system can remain in a high-entropy equilibrium state for an extended period before collapsing entirely.

This dynamic corresponds to the process defined as Scenario C: Systemic Pruning in the HK model. The system does not disappear entirely; however, the institutional structure fragments and transforms into smaller-scale political organisations. The formal dissolution of the Western Roman Empire in 476 CE is the symbolic endpoint of this protracted transformation process.

Implications for the Model

The Roman example supports three important projections of the HK model:

11. Complexity threshold. When institutional systems exceed a certain complexity threshold, marginal returns decline rapidly.
12. Multidimensional entropy. Institutional collapse generally originates not from a single factor, but from the combination of institutional, economic, and epistemic entropy.
13. Prolonged high-Φ regime. Civilisations can sustain their existence under $\Phi > 1$ for extended periods; however, this condition renders the system progressively more fragile.

These implications demonstrate that the HK model can also be employed in the analysis of the modern global system. In the contemporary world, population growth, data production, and the densification of financial networks are rapidly increasing the N variable, while institutional adaptive capacity is not growing at the same pace. This suggests that the global system may be approaching the critical Φ threshold observed in historical civilisations.

4.2. Ancient Greece (800–323 BCE)

Ancient Greek civilisation presents a particularly suitable historical example for examining the relationship between scale, institutional capacity, and epistemic production within the HK

model. The Greek world consisted of a polycentric political structure formed by independent city-states (poleis) rather than large-scale centralised empires. This institutional structure led to the structural limitation of quantitative growth (N), while simultaneously allowing intellectual and cultural production to flourish at an extraordinary level.

When the HK model's key indicator $\Phi = N/Q$ is applied, the development of Ancient Greek civilisation can be approximately interpreted across three phases:

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Rise phase:                          Φ ≈ 0.25 – 0.50
Peak phase:                          Φ ≈ 0.70 – 1.10
Systemic rupture/fragmentation: Φ > 1.10
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These values are not quantitative measurements, but analytical estimates made on the basis of historical data and the comparative analysis of institutional structures.

The Polis System and the Scale Constraint ($\Phi < 1$)

The polis system that emerged in the Greek world from the 8th century BCE can be interpreted in the HK model as a scale-constraining institutional structure. The poleis were typically political units that did not exceed several tens of thousands in population, and political decision-making processes were grounded in citizen participation.

This institutional structure produced several important consequences:

- political decision-making took place at a local scale
- social trust and citizenship identity were strong
- intellectual production flourished within a culture of public deliberation

Under these conditions, while quantitative growth remained limited, institutional capacity could be maintained at a high level. In the HK model, this corresponds to a systemic balance in which the Φ ratio remains low. The cultural production that emerged during this period dramatically elevated the epistemic capacity of the Greek world.

The Persian Wars and Systemic Expansion ($\Phi \rightarrow 1$)

The Persian Wars of the early 5th century BCE significantly transformed the political and military structure of the Greek world. Victory over the Persian Empire led in particular to Athens acquiring immense prestige in the Aegean world.

The Delian League, established in the aftermath of this victory, emerged initially as a defensive alliance; however, it progressively transformed into a hegemonic structure centred on Athens. This process caused a rapid acceleration of quantitative expansion in the Greek world.

From the perspective of the HK model, this development can be interpreted as follows:

- Athenian economic and military power increased.
- Maritime trade and colonisation expanded.
- The alliance system progressively transformed into an empire-like structure.

These developments led to a rapid increase in the N variable. However, institutional capacity could not expand at the same pace. The system therefore approached the threshold $\Phi \rightarrow 1$. The intellectual and cultural explosion that Athens experienced during this period (the Age of Pericles) can be interpreted within the HK model as a typical peak phase.

The Peloponnesian War and Systemic Entropy (Φ > 1)

The most conspicuous indicator of systemic rupture in the Greek world is the Peloponnesian War (431–404 BCE). This prolonged conflict between Athens and Sparta was not merely a military confrontation between two city-states; it was a systemic crisis that disrupted the institutional equilibrium of the Greek world.

This war clearly demonstrates several critical dynamics predicted by the HK model:

Institutional entropy: Throughout the course of the war, the alliance systems between poleis changed continuously and political instability increased. Internal conflicts between democratic and oligarchic factions gave rise to civil wars in numerous cities. This situation can be interpreted in the HK model as an increase in the δ parameter.

Energy and economic entropy: The protracted war weakened economic production and disrupted trade networks. Athens' economy, dependent on maritime trade, was severely weakened towards the end of the conflict.

Epistemic rupture: The Peloponnesian War also generated a crisis of political trust in the Greek world. As evident in Thucydides' writings, the war gravely weakened confidence in political institutions.

The Rise of Macedon and Systemic Pruning

Following the Peloponnesian War, the Greek world remained in a state of political fragmentation for an extended period. This weakness facilitated the subjugation of the region by the rising Kingdom of Macedon to the north. The conquests of the Macedonian king Philip II, and subsequently of Alexander the Great, brought an end to the political independence of the Greek world. Yet this process also gave rise to the birth of a new civilisational form: the Hellenistic world.

From the perspective of the HK model, this development can be interpreted as systemic pruning. The political independence of the polis system was extinguished; however, Greek culture and epistemic production were disseminated across a wider geography. This confirms an important projection of the HK model: systemic collapse most often signifies structural transformation rather than complete annihilation.

Ancient Greece from the HK Model Perspective

The Ancient Greek example lends strong support to three core conclusions of the HK model:

14. The protective effect of the scale constraint. The polis system, by limiting quantitative growth, allowed institutional capacity to be maintained at a high level.
15. The risks of hegemonic expansion. Athens' turn towards an imperial logic increased quantitative pressure and elevated the system's fragility.
16. Critical threshold dynamics. When the Φ ratio exceeded the critical threshold, the system rapidly became susceptible to political fragmentation and external intervention.

FOOTNOTES

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*This section continues the main paper **Social Entropy and Systemic Pruning Theory: Complexity Theory, Historical Cycles and the 21st-Century Civilisational Crisis** (2035–2065). Sections 4.3–6.3.*

4.3. The Abbasid Caliphate (750–1258 CE)

The Abbasid Caliphate constitutes an important historical example for analysing, within the HK model, how a civilisation endowed with exceptionally high epistemic capacity can enter a crisis of institutional coordination. The Abbasid period witnessed a remarkable intellectual ascent in the Islamic world in terms of scientific production, trade networks, and institutional

development; yet the same system experienced severe institutional fragmentation within a few centuries.

In the HK model, the condition of social systems is analysed through the ratio of quantitative pressure (N) to institutional and epistemic capacity (Q):

$$\Phi = N / Q$$

When this ratio exceeds certain thresholds, systemic fragility increases. The historical development of Abbasid civilisation can be interpreted in three principal phases within the HK framework:

Golden Age (high epistemic capacity): $\Phi \approx 0.30 - 0.55$

Period of institutional decline: $\Phi \approx 0.80 - 1.20$

Systemic collapse and external shock: $\Phi > 1.20$

These values are not directly measured quantitative parameters, but analytical estimates derived from the interpretation of historical data in terms of institutional capacity and system scale.

Institutional Reconstitution and Epistemic Ascent ($\Phi < 1$)

The Abbasid Caliphate was established in 750 CE following the end of Umayyad rule, and within a short time constituted the political and cultural centre of the Islamic world. Baghdad, the capital of the caliphate, became one of the largest cities of the medieval world within a matter of decades.

Several important factors account for the high institutional capacity of this period:

- the establishment of centralised bureaucratic governance
- the integration of diverse cultural and intellectual traditions
- the breadth of trade networks
- institutional support for scientific production

The Bayt al-Hikma (House of Wisdom) institution is among the most important examples representing the epistemic capacity of the Abbasid period. This institution functioned not merely as a library, but as the centre of the translation movement and as an academic network in which scientific research was conducted. The translation of Greek, Persian, and Indian scientific texts into Arabic led to significant advances in mathematics, astronomy, medicine, and philosophy. This process can be interpreted in the HK model as a rapid rise in the Q variable.

During this period, although quantitative expansion (N) was increasing, institutional capacity was able to balance this growth. Consequently, the system remained in the condition $\Phi < 1$.

Institutional Complexity and the Crisis of Power Distribution ($\Phi \rightarrow 1$)

The fragility of the Abbasid system began to manifest itself from the 9th century in particular. The management of the caliphate's vast geography required an increasingly complex administrative system. This process brought about several critical institutional transformations.

Transformation of the military system. From the 9th century in particular, Abbasid governance relied upon a professional army composed of Turkish mercenaries. Although this

system initially reinforced central authority, over time it increased the political influence of military elites. In the HK model, this development can be interpreted as a rise in the coordination cost of institutional functioning.

The rise of local power centres. Semi-independent emirates emerged over time across the caliphate's vast territories, weakening the central administration's decision-making capacity. The fragmentation of institutional capacity reduced the effective value of the Q variable.

The fragility of economic networks. The Abbasid economy depended heavily upon long-distance trade networks. However, political fragmentation and security concerns reduced the reliability of trade routes. These developments increased quantitative pressure (N) while preventing institutional capacity from growing at the same pace. As a result, the system approached the threshold $\Phi \rightarrow 1$.

The Contradiction Between Epistemic Power and Institutional Weakness

One of the noteworthy features of Abbasid civilisation is that institutional coordination capacity began to weaken during a period in which scientific production remained at a high level.

This situation supports an important hypothesis of the HK model: epistemic capacity (knowledge production) and institutional capacity do not always develop in parallel. In the Abbasid world, scientific and intellectual production long remained vigorous; yet the fragmentation of political and military institutions weakened the system's overall coordination capacity. This process can be represented in the HK model by a rise in the μ parameter. The μ parameter expresses the erosive effect of epistemic decay on institutional capacity.

Systemic Entropy and the Mongol Shock ($\Phi > 1$)

By the 13th century, the Abbasid Caliphate had become a de facto fragmented political structure. Although the caliph still represented religious authority in institutional terms, political power had largely passed into the hands of regional rulers. This situation can be interpreted in the HK model as a $\Phi > 1$ regime. The quantitative scale of the system remained extensive; however, institutional coordination capacity had been significantly weakened.

The Mongol invasion of 1258 had a devastating effect upon this already fragile system. The fall of Baghdad and the widespread destruction that followed the city marked the symbolic end of the Abbasid Caliphate. From the perspective of the HK model, however, the Mongol invasion should be assessed not as the primary cause but as an accelerating shock. The system had already been in a high-entropy condition for an extended period.

The Abbasid Experience from the HK Model Perspective

The Abbasid Caliphate strongly supports several core projections of the HK model:

1. Epistemic capacity alone is insufficient. Even when scientific and cultural production is at a high level, systemic fragility increases when institutional coordination capacity weakens.
2. Institutional fragmentation accelerates entropy. The empowerment of military elites and the autonomisation of regional rulers weakened the central authority's coordination capacity.

3. External shocks are catastrophic in fragile systems. External shocks such as the Mongol invasion typically cause the rapid collapse of systems that have already become fragile.

From the HK model perspective, this case represents a dynamic in which the δ (institutional decay) and μ (epistemic erosion) parameters rise in concert. This combination demonstrates that even a civilisation endowed with high-quality production can rapidly experience systemic fragmentation once its coordination capacity collapses.

The Abbasid experience thus offers an important analytical lesson for the modern world: even when knowledge production and technological progress are high, systemic stability cannot be guaranteed if institutional coordination capacity weakens.

4.4. The Ottoman Empire (1299–1922)

The Ottoman Empire is a highly instructive historical example for examining, within the HK model, the relationship between institutional adaptive capacity, system scale, and entropy accumulation. This empire, which sustained its existence for approximately six centuries, developed exceptionally powerful institutional mechanisms in its early period and achieved rapid territorial expansion; yet in the modern era, increasing institutional complexity and coordination problems strained the sustainability of the system.

In the HK model, the general condition of the system is represented by the ratio of quantitative pressure to institutional capacity:

$$\Phi = N / Q$$

Here N represents the demographic, geographic, and economic scale of the system, while Q denotes institutional coordination capacity. In addition, the HK model employs two critical parameters:

- δ (institutional entropy coefficient): expresses the rate of increase in bureaucratic deterioration, coordination costs, and institutional inefficiencies.
- ρ (self-repair capacity): represents the capacity for reform, adaptation, and institutional renewal.

The Ottoman historical development can be examined in three main phases within this framework:

$$\text{Rise and institutional consolidation:} \quad \Phi \approx 0.35 - 0.60$$

$$\text{Structural strain and adaptation crisis:} \quad \Phi \approx 0.70 - 1.10$$

$$\text{Systemic dissolution:} \quad \Phi > 1.00$$

These values are not precise measurements, but analytical estimates derived from the interpretation of historical developments in terms of institutional capacity and system scale.

Institutional Innovation and the Rise Phase ($\Phi < 1$)

The early success of the Ottoman state was based to a considerable extent upon a high institutional adaptive capacity. As the empire expanded, it possessed not only military power but also highly functional governance mechanisms.

One of the most important institutional innovations of this period was the devshirme system. Through this system, central governance was able to form a bureaucratic and military elite independent of the local aristocracy, significantly limiting the conflicts of interest that might arise between central authority and local power centres. This mechanism can be interpreted in the HK model as an institutional structure in which the ρ parameter is high: the system continually produced new administrative and military cadres from within its own ranks.

In the same period, the kanunname (law-code) system also served as an important instrument strengthening institutional capacity. The kanunnames standardised local governance practices, facilitating administrative coordination.

Thanks to these institutional arrangements, the Ottoman state achieved rapid geographic expansion while institutional capacity was able to balance this growth over an extended period. The system therefore remained in the condition $\Phi < 1$.

The Rise of Institutional Complexity ($\Phi \to 1$)

From the late 16th century, the Ottoman system faced growing institutional complexity. The vast geography of the empire and its increasing population significantly raised the coordination costs of central governance. This process was accelerated by several critical developments.

The weakening of the devshirme system. Over time the devshirme system lost its former effectiveness and new military elite groups emerged. The increasing political influence of the Janissary Corps in particular created pressure on central authority. This development can be interpreted in the HK model as a rise in the δ parameter.

Pressure on the fiscal system. Prolonged wars and the reorientation of global trade networks strained the Ottoman fiscal system. The emergence of new economic centres competing with Europe weakened the empire's traditional revenue sources, reducing the system's energy and resource flows.

The rise of local power centres. In the 18th century, the growing power of the ayan system weakened central authority's control at the local level, constituting a factor that diminished institutional coordination capacity.

When these dynamics are considered together, it becomes apparent that the Ottoman system was progressively transforming into a structure approaching the threshold $\Phi \to 1$.

The Tanzimat Reforms and the Effort at Adaptation

In the 19th century, Ottoman statesmen recognised the structural problems of the system and undertook comprehensive reform initiatives. The Tanzimat Edict proclaimed in 1839 and the reforms that followed aimed at modernising the structure of the state. The principal objectives of these reforms were:

- strengthening the centralised bureaucratic structure
- modernising legal arrangements
- restructuring the fiscal system
- modernisation of military institutions

From the perspective of the HK model, the Tanzimat reforms can be evaluated as a conscious adaptation effort aimed at increasing the ρ parameter. The state sought to raise the system's self-repair capacity. However, the impact of these reforms remained limited. The most

important reason for this was that the reforms were unable to coalesce into a multi-dimensional coordination strategy.

Systemic Dissolution ($\Phi > 1$)

By the end of the 19th century, the Ottoman system was under severe institutional strain. Governing the empire's vast geography became increasingly difficult, and the balance of power within the international system also shifted against the Ottomans. This process was accelerated by several critical dynamics:

- the rise of nationalist movements
- increasing economic dependency
- intensifying military competition
- international financial pressures

These factors further weakened the system's institutional capacity, causing the Φ ratio to exceed the critical threshold. As a result, the Ottoman Empire rapidly fragmented at the beginning of the 20th century and came to a complete end following the First World War.

The Ottoman Experience from the HK Model Perspective

The Ottoman example strongly supports several important conclusions of the HK model:

4. Institutional innovation can sustain long-term resilience. The early Ottoman institutions rendered the system's expansion sustainable over an extended period.
5. Institutional entropy accumulation is a slow but powerful process. Bureaucratic complexity and the empowerment of military elites eroded the system's coordination capacity over time.
6. Reforms must be multi-dimensional. Although the Tanzimat reforms sought to increase self-repair capacity, they were insufficient to reduce institutional entropy.

From the perspective of the HK model, this case points to an important policy implication: single-dimensional reform strategies are insufficient for resolving systemic crises. For institutional adaptation to succeed, two conditions must be realised simultaneously:

- raising self-repair capacity ($\rho \uparrow$)
- reducing institutional entropy ($\delta \downarrow$)

The Ottoman experience demonstrates that when these two parameters cannot be managed simultaneously, reform efforts may have only limited impact. This finding constitutes an important analytical lesson for contemporary policy-makers: institutional reforms confined solely to the creation of new institutions are insufficient; the coordination costs and institutional entropy accumulated in the existing system must also be reduced.

4.5. Maya Civilisation (250–900 CE)

Maya civilisation constitutes a powerful historical case for examining, within the HK model, the concepts of ecological limits and systemic fragility. Maya cities were high-density urban centres built upon complex agricultural systems and water management networks. These structures increased the qualitative capacity (Q) of the civilisation and supported knowledge

production, social organisation, and cultural output. However, the pressure of quantitative growth (N) against ecological limits reveals the critical dynamics of the HK model.

In the HK model, the condition of a civilisation is analysed through the ratio $\Phi = N/Q$. The historical processes of Maya civilisation can be classified within this framework as follows:

$$\text{Peak Period:} \quad \Phi \approx 0.50 - 0.90$$

$$\text{Systemic Collapse:} \quad \Phi > 1.20$$

These values have been determined not by quantitative measurement, but through the analytical interpretation of institutional, ecological, and demographic data in terms of HK parameters.

Peak Period ($\Phi < 1$)

Between 250 and 700 CE, Maya civilisation developed rapid growth and social organisation. During this period:

- Cities were constructed with dense populations and complex social structures.
- Terraces, canals, and reservoirs were developed to increase agricultural productivity.
- Science, astronomy, and writing systems reached their apex.

In the HK model this corresponds to a high Q value, while N has not yet strained the limits of ecological capacity. The system possesses the institutional and technical capacity to meet entropic pressures.

The Accumulation of Institutional and Ecological Pressures ($\Phi \rightarrow 1$)

From the 700s CE, institutional and environmental pressures began to manifest in Maya cities. In particular:

- Deforestation and soil depletion reduced agricultural productivity.
- Inter-city competition and warfare diverted agricultural investment towards military expenditure.
- Maintenance of water storage and irrigation systems was neglected.

These accumulations can be represented in the HK model through the α parameter (ecological entropy) and the δ parameter (institutional decay). As the N value increases, the effective capacity of Q diminishes and the Φ ratio approaches critical thresholds.

Systemic Collapse and the Ecological Limit ($\Phi > 1.20$)

The severe drought of 800–900 CE served as a triggering factor that accelerated the collapse of Maya civilisation; however, according to the HK model, the fundamental cause of the collapse is the systemic and ecological entropy accumulated prior to this point. The dynamics observed during this process are as follows:

7. Exceedance of Q's carrying capacity: Drought strained the limited capacity of agricultural infrastructure and intensified inter-city resource competition. In the HK model, this can be represented by a decline in Q as conditioned by ecological capacity.
8. Interaction of N and ecological entropy: N is represented by population density and urban growth. Through the α parameter, entropy accumulation exerts a negative effect on Q via resource depletion and ecological destruction.

9. Insufficiency of institutional adaptation: Inter-city competition and the absence of central coordination caused a rise in the δ parameter. As institutional capacity (Q) declined while quantitative pressure (N) remained high, Φ exceeded 1.20.

As a result, Maya cities underwent successive depopulations and urban collapses. From the perspective of the HK model, this collapse is a systemic rupture produced through the ecological entropy channel.

The Maya Experience from the HK Model Perspective

Maya civilisation supports three critical hypotheses of the HK model:

10. Ecological limits determine Q's carrying capacity: The interaction between human-made and natural systems constrains the sustainability of civilisations.
11. N quantitative pressure increases the system's fragility: Intense population and urban density strain institutional and ecological capacity.
12. External shocks are triggers, not causes: Environmental shocks such as drought do not lead to collapse without accumulated entropy and institutional insufficiency.

This mechanism is reoperating in the contemporary context of the climate crisis and ecological limits. In the modern global system, the overexploitation of energy, water, and biological resources is creating an analogous entropic pressure through the α parameter. The Maya experience therefore constitutes a critical warning for contemporary policy-makers with regard to sustainable management and systemic resilience.

4.6. The British Empire (1700–1945)

`Rise: Φ ≈ 0.40–0.70  |  Peak: Φ ≈ 0.85–1.05  |  Collapse: Φ > 1.05`

4.6.1. Historical Dynamics

The British Empire systematically increased quantitative pressure (N) through global trade, colonisation, and the Industrial Revolution during the late 18th and throughout the 19th centuries. Maritime supremacy, the colonial network, and mechanisms of international financial control caused N to rise rapidly across both economic and political dimensions. Institutional and scientific capacity (Q) was, however, maintained at a high level:

- Financial system stability was secured through the Bank of England and powerful central institutions.
- The engineering and manufacturing capacity emerging from the Industrial Revolution elevated Q.
- The colonial bureaucracy and scientific research infrastructure strengthened the system's self-repair capacity in terms of knowledge production and institutional functionality.

During this process, entropic pressures (δ) were at moderate levels; military expansion and colonial administration created coordination deficiencies and local resistance in certain regions.

4.6.2. HK Model Perspective

Within the Cell Cluster (HK) model framework, the rise and decline of the British Empire can be interpreted through the following parameters:

- Self-repair capacity (ρ): High; institutional reforms and central financial mechanisms supported Q.
- Entropy coefficient (δ): Medium; military and colonial expansion created coordination problems in some sub-systems.
- Critical threshold Φ: In the peak period, Φ approached ≈ 1, increasing the system's fragility.

In this context: δ increased over time with global conflicts, economic crises, and fiscal pressures. The high level of Q delayed Φ from reaching the critical threshold; however, the Second World War and colonial losses raised δ, causing the system to enter a process of systemic pruning consistent with Scenario C/D.

4.6.3. Systemic Pruning and Academic Implications

- Systemic Pruning: The post-World War II economic restructuring, colonial losses, and financial shocks show parallels with Scenario C/D in the HK model.
- Φ Dynamics: When high N and high Q are sustained together, the system exhibits high short-term efficiency; however, exceeding the Φ critical threshold reveals an inevitable long-term collapse risk.
- Academic Contribution: The British Empire provides a classic case confirming the HK model's explanatory power regarding the sustainability of high Q alongside high N and the violation of Φ critical thresholds.

4.6.4. Theoretical Depth and Logical Consistency

13. Complexity and marginal returns: Tainter's diminishing marginal returns mechanism has been integrated into the HK model in the context of rising δ alongside the increase in N.
14. Energy and information entropy: Although the Industrial Revolution increased knowledge production, energy and resource pressure directly contributed to Φ.
15. Critical threshold Φ_c: Φ_c > 1 has been taken as the reference point in the model as the quantitative counterpart of systemic fragility and collapse scenarios.

4.7. The Soviet Union (1917–1991)

Rise: Φ ≈ 0.35–0.60 | Peak: Φ ≈ 0.80–1.10 | Collapse: Φ > 1.10

4.7.1. Historical Dynamics

The Soviet Union rapidly increased quantitative pressure (N) in a short period through centralised planning, rapid industrialisation, and collectivisation policies. Large-scale infrastructure projects implemented during 1920–1950, the collectivisation of agriculture, and heavy industry investments led to a rapid rise in N.

- Q (Qualitative Capacity): Attempts were made to increase it through education, scientific research, and military capacity.

- Problematic Factors: Planning errors, bureaucratic rigidity, and the limited flexibility of decision-making led to a rise in the δ and μ parameters.

In this context, the Φ dynamic in the Soviet Union—characterised by high N against limited Q and rising entropy coefficients—clearly approached critical thresholds.

4.7.2. HK Model Interpretation

- Self-repair capacity (ρ): Limited; centralised planning and the single-party system were unable to adequately support Q's self-repair and adaptive capacity.
- Entropy coefficient (δ) and information decay parameter (μ): High; the flexibility of decision-making mechanisms was limited, and bureaucratic density alongside information entropy was growing.
- Φ Dynamics: In the peak period, Φ approached ≈ 1, increasing the system's fragility. The Afghan intervention, economic stagnation, and technological backwardness caused a sudden rise in δ and μ.

The μ parameter represents the lack of information and coordination in centralised planning and bureaucratic decision mechanisms. δ increases over time with economic shocks, military expenditure, and political pressures.

4.7.3. Systemic Pruning and Collapse

- The dissolution of the late 1980s and 1991 represents, according to the HK model, a chronic $\Phi > 1$ collapse.
- This process demonstrates that the system collapsed irreversibly due to the combination of insufficient Q against high N and rising δ/μ.
- A process of systemic pruning consistent with Scenario C, produced by the combined effect of entropic and coordination deficiencies, occurred.

4.7.4. Academic Implications

The Soviet Union example confirms that the imbalance between high quantitative pressure (N) and limited qualitative capacity (Q) plays a critical role in systemic collapse. Rising entropy parameters (δ, μ) are the mathematical counterpart of deficiencies in information flow and bureaucratic coordination, and accelerate the approach of Φ towards the critical threshold. The HK model, by converting historical data into quantitative parameters, provides a powerful instrument for the fragility analysis of political and economic systems. From a modern civilisational perspective, systems incorporating high N and limited Q carry a chronic collapse risk in the absence of pre-emptive systemic pruning or interventions.

4.8. Comparative HK Table and Cyclical Patterns

This concluding finding also carries an important analytical warning for the modern global system. In today's world, while factors such as data production, population growth, and economic concentration are rapidly increasing the N variable, institutional adaptive capacity is not developing at the same pace. This suggests that the global system may also be approaching the critical Φ threshold observed in historical examples.

Table 5. Comparative HK Analysis — Five Civilisations

Civilisation	Rise Φ	Peak Φ	Collapse Φ	Dominant HK Mechanism	Contemporary Parallel
Rome (509 BCE–476 CE)	0.40–0.65	0.80–1.00	>1.00	Institutional entropy (δ↑)	G-20 coordination failure
Greece (800–323 BCE)	0.25–0.50	0.70–1.10	>1.10	Scale overreach	Global governance gap
Abbasid (750–1258 CE)	0.30–0.55	0.80–1.20	>1.20	Coordination collapse (δ, μ↑)	Globally distributed systems
Ottoman (1299–1922)	0.35–0.60	0.70–1.10	>1.00	Insufficient intervention (ρ alone↑)	Fragmented climate agreements
Maya (250–900 CE)	0.50–0.70	0.80–1.20	>1.20	Ecological entropy (α↑)	Climate crisis + biological destruction

The comparative analysis reveals a critical finding: the 21st-century global system is simultaneously exhibiting all the mechanisms that led to the collapse of five distinct civilisations in the past.

Table 5b. Extended Comparative HK Analysis — Seven Civilisations

Civilisation	Period	Φ Rise	Φ Peak	Φ Collapse	Self-repair ρ	Entropy δ	Info Decay μ	N/Q Dynamic	Critical Notes
Roman Empire	509 BCE – 476 CE	0.40 – 0.65	0.80 – 1.00	>1.00	Medium	Medium	Medium	N begins to surpass Q rapidly	Diminishing marginal returns & institutional entropy accumulation; 200+ yr decline
Ancient Greece	800–323 BCE	0.25 – 0.50	0.70 – 1.10	>1.10	High	Medium	Medium	Polis scale constrains Q	N exceeding Q capacity triggers post-peak collapse
Abbasid Caliphate	750–1258 CE	0.30 – 0.55	0.80 – 1.20	>1.20	Medium	High	High	Trade & military inflation raises N	δ and μ rise; even knowledge-intensive systems collapse faster
Ottoman Empire	1299–1922	0.35 – 0.60	0.70 – 1.10	>1.00	High	Medium	Medium	Raising ρ without	Tanzimat reforms one-

								reducing δ insufficient	dimensional; policy lesson: systemic adaptation required
Maya Civilisation	250–900 CE	–	0.50 – 0.90	>1.20	Medium	High	Medium	Ecological pressure α constrains Q	Ecological entropy as collapse mechanism; climate crisis analogous today
British Empire	1700–1945	0.40 – 0.70	0.85 – 1.05	>1.05	High	Medium	Medium	Industrial & colonial network raises N	High Q with high N sustainable; Φ_c breach post-WWII collapse risk
Soviet Union	1917–1991	0.35 – 0.60	0.80 – 1.10	>1.10	Limited	High	High	Planning & collectivisation raise N rapidly	Insufficient Q and rising δ/μ trigger rapid, irreversible collapse

Table Notes

16. Φ (critical threshold): The HK parameter measuring the system's resilience as the ratio of quantitative pressure to qualitative capacity. Φ > 1 represents collapse risk.
17. ρ (self-repair capacity): The ability of institutional and social systems to self-stabilise and adapt.
18. δ (entropy coefficient): Represents the system's coordination deficiency alongside energy and resource pressures.
19. μ (information decay): The effect on the system of distortions in centralised planning, bureaucracy, or information flows.
20. N/Q Dynamic: Indicates how much quantitative pressure (N) strains qualitative capacity (Q); directly affects the critical threshold Φ.

V. SOCIAL ENTROPY THEORY: THE ENERGY AND INFORMATION IMBALANCE OF MODERN CIVILISATION

Social entropy provides the fundamental explanatory mechanism of the systemic fragilities confronting modern civilisations. Drawing on the HK model and historical analyses, social

entropy can be examined in three interconnected layers: energy entropy, information entropy, and institutional entropy. These layers form a vicious cycle that feeds upon itself in both theoretical and mathematical frameworks.

5.1. Thermodynamic Foundations

Energy Entropy

Georgescu-Roegen (1971) emphasised the irreversible energy transformations inherent in economic production. Every act of production and consumption reduces the available energy in the system and increases its entropy. In the HK model context, energy entropy (E_energy) is defined as:

$$E_energy(t) = \int \dot{R}(t)\, dt \,/\, E_0$$

Here $\dot{R}(t)$ represents the total energy consumption of economic and social activities, and E_0 the system's initial energy endowment. As energy entropy increases, the system's capacity to balance the Q and N variables diminishes.

The principal sources of energy entropy in modern civilisation are:

- fossil fuel consumption and carbon emissions
- the energy intensity of global logistics and transportation networks
- the energy consumption of digital infrastructure and data centres

Energy entropy is a critical component contributing to a rise in the Φ ratio within the HK model.

Information Entropy

Shannon's (1948) information theory demonstrates that as noise in communication increases, average information transmission decreases. For social systems, this means that although knowledge production increases quantitatively, the capacity for meaning and coordination diminishes. Information entropy (E_info) can be represented in the HK model context as follows:

$$E_info = -\Sigma\, p_i \log(p_i) + \eta \cdot N_dig$$

In this formula:

- p_i: the relative weight of the different information channels in the system;
- N_dig: related to the quantitative pressure of digital data volume and the platform economy;
- η: the coefficient of digital concentration's contribution to information erosion.

Information entropy is increasing rapidly in modern civilisations, particularly with digitalisation and data concentration. The growth of N_dig erodes Q's effective capacity, causing the Φ ratio to approach the critical threshold.

Institutional Entropy

Institutional entropy refers to the erosion of a society's or civilisation's coordination and adaptive capacity. Bureaucratic over-complexity, decision-making delays, and local/central power imbalances can be represented by the δ parameter.

In the HK model, institutional entropy increases through the positive feedback effect of energy and information entropy:

$$\delta(t) = f(E_energy, E_info, N)$$

This function quantitatively expresses the erosive effect of increasing energy and information entropy on institutional efficiency. For example:

- Rising energy entropy reduces infrastructure maintenance and repair capacity → δ ↑
- Rising information entropy reduces the effectiveness of policy and planning decisions → δ ↑
- Rising quantitative pressure (N) raises coordination costs → δ ↑

These three layers form a social entropy vicious cycle that feeds upon itself:

$$E_energy \uparrow \rightarrow \delta \uparrow \rightarrow E_info \uparrow \rightarrow Q \downarrow \rightarrow \Phi \uparrow$$

Modern Civilisation Perspective

Modern civilisation exhibits several distinctive features from the standpoint of social entropy:

21. Energy intensity and global connectivity: Global supply chains and high-technology infrastructure rapidly increase energy entropy.
22. Digital data and algorithmic concentration: Platform economies expand digital information volume; N_dig and algorithmic entropy (the γ parameter) accelerate the increase in Φ.
23. Rising costs of institutional adaptation: States and institutions struggle to manage increasing complexity; single-dimensional reforms (raising ρ alone) are insufficient to reduce δ.

Table 6. Relationship Between Social Entropy Layers and HK Model Parameters

Layer	HK Parameter(s)	Effect
Energy Entropy	E_energy	Constrains Q capacity; raises Φ
Information Entropy	N_dig, γ	Erodes Q's depth of meaning; elevates δ
Institutional Entropy	δ	Reduces system's self-repair and coordination capacity
Cultural Entropy	E_culture, α_c	Degrades Q_eff; impairs long-term adaptive capacity
Scientific Entropy	E_science, γ	Weakens knowledge-generation and self-repair capacity

5.2. Systemic Effects: Complexity, Marginal Returns, and Chronic Φ > 1

Complexity and Marginal Returns: The Tainter Perspective

Tainter (1988), in his study of the collapse processes of historical civilisations, demonstrated that as complexity increases, the benefit provided by each additional unit of complexity diminishes; when a threshold is exceeded, marginal returns turn negative. From the HK model perspective, this diminishing marginal return can be expressed as:

$$dR/dC < 0 \quad (\text{for } C \geq C_{threshold})$$

where R denotes institutional and social 'welfare' or effective output, and C denotes system complexity (bureaucracy, technological infrastructure, social organisation). This approach is directly related to the Φ ratio: when quantitative pressure (N) surpasses qualitative capacity (Q), marginal utility production declines and systemic fragility increases.

Chronic Φ > 1 and Contemporary Financial Systems

According to IMF data, by 2026 the ratio of global public debt to GDP has exceeded 100%. Within the HK framework, this situation can be interpreted as a chronicisation of Φ > 1:

- N: global debt stock and financial burden
- Q: institutional capacity and economic governance capacity

This chronic Φ > 1 condition is reducing the system's marginal returns and diminishing its capacity for sustainable growth.

The Systemic Effect of Energy and Information Entropy

Systemic fragility is not limited to financial pressures; energy and information entropy are also factors that increase the Φ ratio:

24. Energy entropy (E_energy): Rising energy costs and fossil fuel dependency constrain production capacity.
25. Information entropy (E_info): Digital data volume, algorithmic concentration, and communication noise reduce the quality of institutional decision-making.

In the HK model, the chronic Φ > 1 condition is continuously fed by a positive feedback loop of these three entropy layers:

$$d\Phi/dt = \dot{N}/Q - N \cdot \dot{Q}/Q^2$$

If δ is persistently greater than ρ, the system chronically sustains the Φ > 1 condition.

Debt, N, and Systemic Fragility

The global debt stock is estimated at approximately 305 trillion USD, against a global GDP of approximately 90 trillion USD. Under these conditions, Φ_debt ≈ 3.39 (scaled normalised value of N/Q) can be calculated. The system exhibits a condition analogous to the chronic high Φ and diminishing marginal returns observed in historical civilisations:

- The marginal benefit of additional financial and institutional investment for economic growth is low.
- The system faces structural fragility and a 'high Φ trap'.

Logical and Academic Implications

26. The complexity threshold is a critical parameter: every system loses its capacity for marginal utility production beyond a certain N/Q ratio.
27. A chronic $\Phi > 1$ condition indicates long-term fragility: factors such as financial debt, energy dependency, and digital data concentration support this condition.
28. Policy recommendation: for sustainable governance, it is not sufficient merely to raise self-repair capacity; entropy sources must also be actively reduced: $\delta \downarrow$, N controlled, Q strengthened.
29. Integration with the HK model: all these systemic effects can be analysed through numerical simulations using the HK model's parameters (ρ, δ, N, Q) and the Φ ratio.

5.3. The Erosion of Cultural and Scientific Values

The social entropy dynamics of modern civilisations are not confined to economic and energy-information layers; the erosion of cultural and scientific values is a critical mechanism that increases long-term systemic fragility. Within the HK model and social entropy framework, this erosion directly affects Q's qualitative capacity.

Collective Memory and Cultural Entropy

Assmann (2008) notes that collective memory preserves the historical continuity and identity of societies. In this context, collective memory is preserved through institutional memory, archives, education systems, and cultural narratives. The erosion of these structures under the pressure of consumer culture produces cultural entropy (E_culture) and directly exerts an erosive effect on Q.

Mathematically:

$$Q_{eff}(t) = Q(t) \cdot (1 - \alpha_c \cdot E_{culture}(t))$$

where Q_eff(t) is effective qualitative capacity, α_c is the weight of cultural entropy on Q, and E_culture(t) is the rate of erosion of cultural values. This formula demonstrates that the preservation of collective narratives and cultural values strengthens social capacity, while neglect increases the system's fragility.

Academic Institutions and Scientific Entropy

Collini (2012) emphasises that the integration of universities into market logic can reduce the capacity for critical thinking and scientific research. This situation directly contributes to the information entropy (E_info) layer in the HK model. When research and critical thinking capacity weaken, Q decreases and the Φ ratio rises.

Within the HK model framework, scientific entropy (E_science) is defined as:

$$E_{science} = \gamma \cdot (R_{market} / R_{total})$$

where R_market is market-oriented research expenditure, R_total is total research capacity, and γ is the coefficient affecting knowledge entropy on Q. Rising E_science limits the system's long-term adaptive and self-repair capacity.

Interactive Fragility of Cultural and Scientific Values

Cultural and scientific erosion forms a positive feedback loop with other social entropy layers:

30. Consumer culture and market-oriented scientific approaches → E_culture ↑, E_science ↑
31. Q's qualitative capacity diminishes → Φ ↑
32. Rising Φ increases quantitative pressure (N) and amplifies effects on energy/information entropy → δ ↑

Policy Implications from the HK Model Perspective

33. The preservation of cultural heritage is a strategic intervention that increases civilisation's Q capacity.
34. The independence of universities and scientific institutions is critically important for managing information entropy and reducing Φ.
35. The neglect of cultural and scientific layers, when combined with energy and digital entropy pressures, accelerates modern civilisation's descent into the chronic $\Phi > 1$ trap.

5.4. Numerical Integration of Social Entropy into the HK Model and Monte Carlo Scenarios

For understanding the complex dynamics of modern civilisation, the numerical integration of energy, information, institutional, and cultural entropy layers with the HK model is a critical step. This section addresses the association of each entropy layer with HK parameters, the stochastic extension of the model, and the production of projections through Monte Carlo scenarios.

5.4.1. Integration of Entropy Layers into the HK Model

The fundamental variables defining the condition of a civilisation in the HK model are summarised by $\Phi = N/Q$. These variables are updated by the effect of four social entropy layers:

36. Energy entropy (E_energy): Constrains Q's maximum capacity; raises δ.
37. Information entropy (E_info): Digital data density and algorithmic excess reduce Q's depth of meaning; elevate δ.
38. Institutional entropy (δ): Complex bureaucracy and coordination deficiencies lead to an increase in Φ.
39. Cultural and scientific entropy (E_culture, E_science): The erosion of collective memory and critical thinking reduces Q's long-term self-repair capacity.

These interactions can be expressed in the HK model through extended stochastic differential equations (SDE):

```
dQ = [ρ·Q(1−Q) − δ(E,I,N)·Q − μ·S·Q] dt + σ_Q·dW_Q
dN = [r_N·N·(1−N/K) + σ·S·N] dt + σ_N·dW_N
```

This formulation provides the means to simulate how modern civilisation will evolve under both deterministic and stochastic pressures.

5.4.2. Monte Carlo Scenarios

The model can be run in four principal scenarios for the 2035–2065 period:

- Scenario A (Q_0=0.90; N_0=0.80; σ_Q=0.01; σ_N=0.02): Stable equilibrium; entropy increase limited.
- Scenario B (Q_0=0.85; N_0=1.45; σ_Q=0.03; σ_N=0.05): Critical transition; digital and debt pressures effective.
- Scenario C (Q_0=0.80; N_0=2.00; σ_Q=0.05; σ_N=0.10): Systemic pruning; energy and cultural entropy high.
- Scenario D (Q_0=0.80; N_0=2.00; σ_Q=0.04; σ_N=0.08): Intervention-assisted recovery; policy interventions simulated.

Through Monte Carlo simulations, the Φ time series, Q and N distributions, and δ evolution for each scenario are obtained. This method offers probability-based risk analysis and the capacity to test policy effects.

5.4.3. Analysis of Model Outputs

Through the HK model + Monte Carlo integration:

40. Identification of critical thresholds: Under which conditions Φ exceeds 1.2 or 1.5 is determined.
41. Decomposition of systemic fragility sources: The separate and interactive effects of energy, information, cultural, and institutional entropy are measured.
42. Evaluation of policy scenarios: δ-reducing interventions, ρ-increasing reforms, and N-control strategies are simulated, and long-term Q and Φ projections are examined.

Figure 4 — Monte Carlo Scenario Projections

Φ(t) time series for Scenarios A–D (n=1,000 iterations). Shaded bands represent 95% confidence intervals. Q and N distributions at t = 0, 100, 200, 300 steps. [Figure generated from Python/Monte Carlo output.]

Figure 4. Monte Carlo Scenario Projections — Φ(t) time series with 95% confidence bands; Scenarios A–D (2035–2065).

5.4.4. Projections for Modern Civilisation

- The 2035–2065 period is projected as a timeframe in which systemic entropy approaches the critical threshold.
- The chronic Φ > 1 condition is supported by energy, debt, and digital pressures.
- The preservation of cultural and scientific values is a critical policy instrument for sustaining Q capacity.
- Monte Carlo outputs permit the testing of the effectiveness of policy interventions within a probability-based framework, enhancing the predictive power of the HK model.

VI. LITERATURE REVIEW

For understanding the transformation processes of modern civilisations, the literature in the fields of complexity theory and rupture models provides a critical foundation. This section deepens, in both theoretical and mathematical terms, how the literature can be related to the HK model and social entropy perspective.

6.1. Complex Systems and Rupture Theory

Complex Adaptive Systems

Holland (1995) and Kauffman (1993) define the fundamental characteristics of complex adaptive systems as follows:

43. Multiple interacting components: System components affect one another through local and global interactions.
44. Emergent order: The system reorganises itself through collective arrangements emerging from the behaviour of individual components.
45. Adaptation and learning: The system responds to environmental changes and learns from past experience.
46. Critical thresholds and marginal returns: As complexity increases, the system's marginal utility production diminishes.

The HK model represents this theoretical framework numerically through system dynamics and entropy layers. As complexity increases, Q's effective capacity diminishes via δ, and the Φ ratio approaches critical thresholds.

The Panarchy Cycle and Ecological Rupture

Gunderson and Holling (2002) defined the four-stage 'panarchy' cycle of ecosystems:

47. Growth (r): Resources are used efficiently; the system gains stability.
48. Conservation (K): The system's structural complexity increases; flexibility begins to decline.
49. Collapse (Ω): Marginal returns turn negative; crisis begins.
50. Reorganisation (α): The system reaches a new equilibrium; adaptation takes place.

The HK model quantifies this cycle through the Φ parameter:

- $\Phi < 1$: growth and conservation
- $\Phi \approx 1$: critical threshold
- $\Phi > 1$: collapse and systemic pruning

This approach combines the analysis of historical civilisations with the simulation of modern digital and economic systems.

Critical Threshold Studies

Lenton et al. (2008) emphasised the importance of tipping points in ecosystems and climate systems. These critical thresholds cause the system to undergo major transformations through minor changes. For global civilisation, factors such as debt, energy, digital pressure, and cultural erosion may cause Φ to exceed these critical thresholds.

Integration of the Literature with the HK Model

51. Complex adaptive systems: represented in the HK model through the interaction of Q and N.
52. Panarchy cycles: the phases of growth, conservation, collapse, and reorganisation of the system are modelled using the Φ ratio and δ coefficient.
53. Critical thresholds: the probability of $\Phi > 1$ or the exceedance of Φ_c is calculated on a probability basis through Monte Carlo simulations.
54. Social entropy layers: energy, information, institutional, cultural, and scientific entropy numerically feeds the complex systems mechanisms in the literature.

Academic and Mathematical Contribution

55. Theoretical Depth: The complexity and critical threshold literature strengthens the philosophical and systemic foundations of the HK model.
56. Mathematical Consistency: The Q, N, and δ variables are consistent with the concepts of marginal returns, panarchy, and tipping points in the literature.
57. Policy and Simulation Link: The projections in the literature are rendered applicable in the HK model through Monte Carlo scenarios and entropy layers.

6.2. Platform Neo-Feudalism and Surveillance Capitalism

For understanding the social, economic, and digital dynamics of 21st-century civilisation, the literature on platform neo-feudalism and surveillance capitalism provides a critical framework. This section re-presents, in an integrated manner with the HK model, a deepened theoretical clarity alongside mathematical and logical consistency.

6.2.1. Surveillance Capitalism and Data Power

Zuboff (2019) defines surveillance capitalism as follows: individuals' behaviours are monitored through continuous data production processes and fed to algorithms for the purpose of creating economic value. This process rapidly increases the digital component of the N variable (N_{dig}) and strains Q's (qualitative capacity) capacity.

Digital pressure and data concentration in the HK model:

$$N_{dig}(t) = P(t) \cdot \lambda \cdot N_{dig}$$

where P(t) represents platform power concentration (market share, user base) and λ represents the algorithmic and platform-based entropy coefficient. This formula mathematically represents digital and economic pressure increasing the Φ ratio and causing the system to approach the critical threshold.

6.2.2. Platform Capitalism and Neo-Feudal Structures

Srnicek (2017) defines platform capitalism as a neo-feudal structure in which traditional capitalist relations of production are combined with algorithmic and digital centralisation. From the HK model perspective:

- Platforms centralise economic and data power → N increases.
- Mechanisms that would increase institutional and cultural capacity are limited → Q remains constant or grows slowly.

- This asymmetric structure causes Φ to remain persistently high.

Rising N_dig and limited Q amplify the system's fragility through a positive feedback loop.

6.2.3. The Attention Economy and Social Harm

Lanier (2018) explains the long-term effects of the attention economy on individuals and society as follows: digital platforms direct individual attention resources through commercial algorithms. This direction weakens social learning and critical thinking capacity → Q decreases. In the HK model, this effect is integrated into the cultural and scientific entropy layer (E_culture, E_science).

Rising digital pressure not only increases economic and social asymmetries, but also reduces social learning and self-repair capacity, leading to the chronicisation of Φ.

6.2.4. Policy and Mathematical Consistency from the HK Model Perspective

58. Digital pressure and algorithmic concentration → N_dig ↑ → Φ ↑ → δ ↑
59. Deficiency in institutional and cultural capacity → Q limited → Φ approaches the critical threshold
60. Positive feedback loop → chronic Φ > 1 condition

These dynamics can be tested numerically through Monte Carlo simulations and extended versions of the HK model.

6.2.5. Academic Implications

- Platform neo-feudalism is a structural factor leading to the chronicisation of Φ in the HK model.
- Surveillance capitalism triggers digital pressure and N_dig growth, increasing social and information entropy.
- The attention economy and cultural erosion reduce Q's effective capacity and amplify long-term fragility.

6.3. Ecological Limits and the Critique of Growth

The literature evaluating the sustainability of modern civilisations increasingly and forcefully demonstrates that economic growth is constrained by biophysical limits. In this context, the ecological limits approach should be treated not merely as an environmental discussion, but as a structural constraint system that determines the long-term stability of civilisations.

6.3.1. Bounded Growth and the World System

Meadows et al. (1972), in Limits to Growth, demonstrated that unlimited economic and population growth is unsustainable due to biophysical limits. Resource consumption and waste production exceed the system's energy and ecological carrying capacity. From the HK model perspective, this is represented by the N/Q ratio approaching the critical threshold Φ_c.

6.3.2. Ecological Thresholds and Planetary Boundaries

Rockström et al. (2009) identified nine planetary boundary systems—including climate change, biodiversity loss, ocean acidification, and the nitrogen cycle—and demonstrated that several of these boundaries have already been crossed. Within the HK model framework, the exceedance of each planetary boundary contributes to a rise in the α (ecological entropy)

parameter, creating a direct erosive effect on Q's capacity. The ecological limit threshold can be expressed as:

$$Q_eco(t) = Q(t) \cdot (1 - \alpha \cdot E_eco(t))$$

where E_eco(t) represents the rate of ecological entropy accumulation, and α is the coefficient of its effect on Q. Ecological limits constitute a critical parameter system that directly reduces Q's carrying capacity in the HK model.

Figure 5 — Ecological Entropy and Q Degradation

Simulated Q eco(t) trajectories under low, medium, and high ecological entropy rates (α = 0.02, 0.06, 0.10). Shaded region indicates Φ > 1 (systemic fragility zone). [Figure generated from Python output.]

Figure 5. Ecological Entropy Degradation of Q — Scenarios by α coefficient; 2035–2065 projection.

[Sections VI.3.2 onwards, and Sections VII–X, continue in the main manuscript.]

*Continuation of sections 6.3.2 – 10.5 | **Social Entropy and Systemic Pruning Theory: 2035–2065***

6.3.2. Ecological Thresholds and Planetary Boundaries

Steffen et al. (2015) advanced the 'planetary boundaries' concept by identifying critical ecological thresholds. These boundaries encompass climate change, biodiversity loss, land-system change, disruptions to the nitrogen and phosphorus cycles, freshwater use, stratospheric ozone depletion, aerosol loading, ocean acidification, and chemical pollution. In the HK model, these boundaries quantitatively express the effect on Q through the ecological entropy coefficient α:

$$Q_eco(t) = Q_0 \cdot e^{(-\alpha \cdot E_eco(t))}$$

This formula demonstrates that the transgression of ecological limits reduces qualitative capacity and thereby increases systemic fragility.

6.3.3. Fair and Bounded Growth

Raworth (2017), in Doughnut Economics, repositions the growth paradigm within a framework of social and ecological limits:

- Lower boundary: fundamental human rights and living standards
- Upper boundary: ecological limits and resource carrying capacity

Integration with the HK model: the lower boundary corresponds to the institutional capacity threshold that guarantees the preservation of Q, while the upper boundary corresponds to the ecological Φ_c threshold that N must not exceed.

6.3.4. Modern Civilisation Perspective and HK Interpretation

1. Energy and resource consumption: fossil fuel and intensive agricultural pressure cause N to rise rapidly.
2. Digital and financial pressures: platform neo-feudalism and the debt burden accelerate Φ's approach to critical thresholds.
3. Cultural and scientific capacity: the decline of Q limits the social capacity for self-repair and adaptation.
4. Ecological entropy: the α parameter and E_eco integrate the transgression of environmental limits into the HK model, quantifying crisis scenarios.

6.4. Political and Institutional Fragility

In evaluating the sustainability of 21st-century civilisation, political and institutional fragility constitutes a critical component for understanding the behaviour of social, economic, and digital systems.

6.4.1. Institutional Resilience and Collapse Mechanisms

Acemoglu and Robinson (2012) emphasise the role of institutional structure in political and economic stability:

- Inclusive institutions: rule of law, property rights, and transparent governance; these strengthen Q's long-term self-repair capacity.
- Extractive institutions: limited power-sharing and power concentration; these raise δ, causing Φ to approach the critical threshold.

$$dQ/dt = \rho \cdot Q \cdot (1 - Q) - \delta(\text{institutional_entropy}) \cdot Q$$

6.4.2. Institutional Fragility and Democratic Challenges

Fukuyama (2014) emphasises the importance of political fragility when measuring states' capacities and institutional effectiveness:

- State capacity: crisis management, resource allocation, infrastructure, and public services
- Institutional trust: society's trust in institutional structures preserves Q and stabilises Φ

$$Q_state(t) = Q_0 \cdot (1 + \rho_state \cdot trust(t)) - \delta_capacity \cdot t$$

6.4.3. The Erosion of Democratic Norms and the Deficit of Tolerance

Levitsky and Ziblatt (2018) demonstrate how the weakening of democratic norms disrupts political stability:

- The transgression of institutional limits on power leads to the incapacitation of checks-and-balances mechanisms.

- In the HK model, this situation is modelled as a process that raises δ through a positive feedback loop: democratic erosion → δ ↑ → Q ↓ → Φ ↑.

6.4.4. Modern Civilisation Perspective

- Rising digital and financial pressures, platform neo-feudalism, and the debt burden increase political and institutional fragility → δ rises.
- In scenarios where institutional fragility is high, Q's qualitative capacity diminishes and the Φ > 1 condition becomes chronic.
- These dynamics can be tested through stochastic simulations and Monte Carlo scenarios using the HK model.

6.4.5. Academic and Political Implications

5. Institutional capacity is directly related to Q's self-repair function and is critical for the sustainability of modern civilisation.
6. Political fragility raises δ and causes Φ to approach the critical threshold; it triggers the chronic Φ > 1 condition.
7. The mathematical formulation of the HK model, by integrating political and institutional fragility, enables scenario-based risk analysis of modern civilisation.

VII. GLOBAL SYSTEM PRESSURES: POPULATION, ENERGY, ECOLOGY AND RESOURCE COMPETITION

Table 7. Global System Pressures — HK Parameter Mapping and Contemporary Indicators

Pressure Domain	Key HK Mechanism	Primary Parameter	Feedback Effect	Contemporary Indicator
Demographic	Rapid N increase outpacing Q growth	N↑, δ↑	Demographic fragility trap	UN population projections; Gini index
Energy Transition	EROI gap creates temporary Φ spike	Q_energy, δ↑	Transition fragility lock-in	IEA EROI data; renewable share
Ecological Limits	Ecological entropy erodes Q carrying capacity	α, E_eco, K_eff	Ecological Φ trap	Planetary Boundaries index; GFN footprint
Financial System	Derivative leverage inflates N_finance	N_finance, δ↑, μ↑	Systemic cascade risk	BIS derivative data; debt/GDP ratio
Platform Neo-Feudalism	Algorithmic entropy erodes Q_eff	γ·P(t)·Q, N_dig↑	Digital lock-in (Scenario E)	HHI ad revenue; DataSphere growth
Political Fragility	ρ decline + δ rise → chronic Φ>1	ρ↓, δ↑	Coordination failure cascade	Freedom House democracy index

7.1. Demographic Pressures

Within the dynamics of the global system, demographic structure is not merely a quantitative variable; it also encompasses a broad sphere of influence extending from economic productivity to political stability, and from technological adaptation to resource consumption. Demography is therefore treated in the HK model as one of the primary determinants of quantitative pressure (N).

7.1.1. Global Demographic Transition and Structural Asymmetry

According to United Nations projections, the world population is expected to continue growing until 2050; however, this growth is not homogeneous. The global system is producing a tripartite demographic divergence:

8. High-fertility regions (Sub-Saharan Africa) → rapid N increase
9. Declining-fertility regions (South Asia, Middle East) → transition dynamics
10. Ageing societies (Europe, East Asia) → N stabilisation or contraction

This asymmetry means that different sub-systems in the HK model have different Φ dynamics. The global system now exhibits not a single equilibrium but a condition of multi-equilibrium instability.

7.1.2. Mathematical Representation of Population Dynamics

Demographic growth is classically expressed through the logistic growth model:

$$dN/dt = r_N \cdot N \cdot (1 - N/K)$$

This formula shows that population growth is determined not only by biological factors, but also by institutional and technological capacity.

7.1.3. Demographic Pressure and Φ Dynamics

$\Phi = N/Q$, the core ratio of the HK model, elevates demographic pressure through two distinct mechanisms:

11. Direct effect: increase in N
12. Indirect effect: pressure on Q (education, infrastructure, labour market)

This condition defines an unsustainable demographic pressure in regions experiencing a youth population boom in particular: $dN/dt > dQ/dt \Rightarrow d\Phi/dt > 0$.

7.1.4. Demographic Transition and Economic Structure

Demographic transformation affects the economic system through three principal channels:

- Labour supply: youth population growth is an advantage in the short term, but carries a risk of unemployment in the long term
- Dependency ratio: the productive population declines in ageing societies
- Human capital: educational attainment determines Q

This relationship can be expressed in the HK model as: Q = f(educational_attainment, infrastructure, institutional_capacity). This formula emphasises that the quality of the population, rather than its size, is determinative.

7.1.5. Demographic Entropy and Social Pressure

Rapid population growth produces entropy in social systems. In the HK model this is related to the δ parameter: δ = f(dN/dt, infrastructure_insufficiency). High demographic pressure weakens social cohesion, increases political instability, and erodes institutional capacity, indirectly causing a decline in Q.

7.1.6. Global Power Balance and Demography

Demography is one of the primary factors reshaping the distribution of power in the international system. Three fundamental structures can be identified from the perspective of the HK model:

- High N but low Q → fragile ascent
- Balanced N and high Q → sustainable power
- Low N and high Q → innovative but demographically vulnerable structure

This tripartite structure is one of the primary mechanisms explaining asymmetric power transitions in the 21st century.

7.1.7. Feedback Loop: The Demographic Pressure Trap

Demographic pressure can form a self-reinforcing cycle:

13. N rises → Φ increases
14. Pressure on Q increases → education and infrastructure prove insufficient
15. Unemployment and social tension rise → δ increases
16. Institutional capacity weakens → Q declines
17. Φ rises further

This cycle is defined in the HK model as the demographic fragility trap.

7.1.8. Academic and Theoretical Contribution

18. Theoretical contribution: integrates demographic transition theory with complexity and system dynamics.
19. Mathematical contribution: explains the relationships among N, Q, and δ through differential equations.
20. Analytical contribution: converts global demographic asymmetry into a measurable fragility indicator.

7.1.9. Conclusion: Demography and Systemic Pressure

From the HK model perspective, demography is one of the most powerful systemic pressures facing modern civilisation. If N growth is not controlled, Φ rises rapidly; if Q cannot be sufficiently increased, the system becomes fragile; and an increase in δ triggers political and social instability.

7.2. Energy Transition Dynamics

The sustainability of global civilisation is directly related to the rate and structural characteristics of the transformation of energy systems. Energy plays a bidirectional role in the HK model: it is simultaneously a growth input that increases quantitative pressure (N) and a primary determinant of qualitative capacity (Q).

7.2.1. Energy Regimes and Historical Lock-In

Energy systems historically produce strong infrastructural path dependency and lock-in effects. Fossil fuel-based systems are bound to capital-intensive infrastructures, have long amortisation periods, and are integrated with institutional and political networks. This renders energy transitions nonlinear and slow processes. From the HK model perspective, this means that Q's rate of transformation is limited.

7.2.2. Energy Supply, EROI, and System Capacity

The effectiveness of energy systems is determined not only by total output, but also by the Energy Return on Investment (EROI). The energy component of Q in the HK model can be expressed as:

$$Q_{energy}(t) = f(EROI(t), production_capacity)$$

EROI, historically high in fossil fuel systems, tends to decline over time. In renewable systems, the initially lower EROI is increasing through technological development. The 'EROI gap' generated during this transition creates temporary pressure on Q and can increase the Φ ratio.

7.2.3. Mathematical Dynamics of Energy Transition

Energy transition can be modelled as competition between two systems, demonstrating why energy transformation occurs not in a linear fashion but in the form of an S-curve (logistic substitution):

$$dF/dt = -r_F \cdot F \cdot (1 - R/K_R)$$

$$dR/dt = r_R \cdot R \cdot (1 - R/K_R)$$

7.2.4. Energy Transition and Φ Dynamics

Energy systems affect the Φ ratio in the HK model through three channels:

21. N increase: energy abundance accelerates economic growth
22. Q increase: efficient energy systems raise productivity
23. δ increase: systemic costs and uncertainty rise during the transition period

During the transition period, Q grows temporarily slowly while N continues to grow; this produces $dN/dt > dQ/dt \Rightarrow d\Phi/dt > 0$, i.e., an increase in systemic pressure.

7.2.5. Energy Entropy and Systemic Cost

Energy transformations are costly, entropy-generating processes. During the transition, infrastructure creates a dual burden (old + new system), supply chains are restructured, and political/economic uncertainty increases. This causes δ to rise and the system to become more fragile.

7.2.6. Geopolitics and Resource Competition

Energy transition is not merely a technological restructuring process, but a geopolitical one as well. As fossil fuel-centred power balances weaken, critical minerals (lithium, cobalt, rare earth elements) become strategically important and energy dependency relations are reshaped. In the HK model, this can be expressed through the external pressure parameter μ: high μ increases the coordination cost of the energy transition and reduces Q's effectiveness.

7.2.7. Feedback Loop: The Energy Transition Trap

24. Energy transformation begins → costs increase
25. Q is temporarily suppressed
26. Economic growth slows → pressure on N increases
27. Political and social tension rises → δ increases
28. Transition process slows → system locks in

This cycle is defined in the HK model as transition fragility.

7.2.8–7.2.10. Systemic Balance, Academic Contribution, and Conclusion

For energy transition to be sustainable, the following conditions are required in the HK model: $dQ_energy/dt \geq dN/dt$ and $\delta(transition) < \rho(adaptation)$. When these conditions are not met, energy transition can transform from a stabilising factor into one that increases fragility.

From the HK model perspective, energy transition is a bidirectional process for modern civilisation: while it strengthens sustainability by increasing Q in the long run, it generates fragility in the short and medium term by increasing δ and μ.

7.3. Ecological Limits

The sustainability of global civilisation is determined not only by economic and technological capacity, but also by the limits that biophysical systems can sustain. The ecological limits approach demonstrates that when the effects of human activity on the Earth system exceed certain critical thresholds, irreversible changes can occur.

7.3.1. Planetary Boundaries and Systemic Thresholds

The planetary boundaries approach defines critical thresholds in fundamental systems such as climate change, biosphere integrity, nitrogen-phosphorus cycles, and land use. When these thresholds are exceeded, the system produces nonlinear responses, feedback mechanisms accelerate, and the system departs from a stable equilibrium. From the HK model perspective, this is expressed as the erosion of Q's carrying capacity:

$$Q_eco(t) = Q_0 \cdot e^{(-\alpha \cdot E_eco(t))}$$

This formula shows that the transgression of ecological limits directly weakens production capacity and system organisation.

7.3.2. Ecological Entropy and Biosphere Erosion

Degradation in ecological systems is treated as entropy production in the HK model. Biodiversity loss in particular is a critical factor that reduces the system's adaptive capacity. An increase in this variable produces a two-directional effect:

29. It reduces Q's effective capacity
30. It produces an amplifying effect on δ (total entropy coefficient): $\delta_{eco} = \gamma \cdot E_{eco}(t)$

This structure demonstrates that ecological degradation is a process that destabilises not only the environment, but also economic and institutional systems.

7.3.3. Critical Thresholds and Nonlinear Dynamics

The most important characteristic of ecological systems is that they exhibit sudden and disproportionate responses when certain thresholds are exceeded. This corresponds to the approach of the Φ ratio to its critical value in the HK model. When ecological limits are transgressed, Q_{eff} declines rapidly and the condition $\Phi = N/Q_{eff} > \Phi_c$ is quickly met, causing the system to become fragile in a short period of time.

7.3.4. Ecological Pressure and Carrying Capacity

In classical growth models, carrying capacity (K) is treated as a constant. However, under ecological limits, K is a dynamic variable:

$$K_{eff}(t) = K_0 \cdot (1 - \alpha \cdot E_{eco}(t))$$

This equation reveals that the transgression of ecological limits constrains not only the environment but directly the dynamics of population and economic growth.

7.3.5. Ecological Collapse Mechanism

The transgression of ecological limits produces a tripartite systemic collapse mechanism:

31. Resource depletion: energy and food systems weaken
32. Capacity decline: Q decreases
33. Entropy increase: δ rises

This structure demonstrates that the effect of ecological degradation on Q is not linear but accelerating in character: $dQ_{eff}/dt = -\alpha \cdot (dE_{eco}/dt) \cdot Q$.

7.3.6. The Global System and Asymmetric Effects

Ecological limits do not affect all regions equally:

- Low-income regions → higher fragility
- High-income regions → higher adaptive capacity
- Middle-income regions → transition fragility

The HK model explains this through different Φ values across sub-systems. This produces second-order effects in the global system such as migration pressure, resource competition, and political instability.

7.3.7. Feedback Loop: The Ecological Fragility Trap

34. Ecological degradation increases → Q declines
35. As Q declines, production pressure increases → N continues to grow
36. As N grows, resource use increases → E_{eco} rises
37. δ increases → system coordination breaks down

38. System becomes more fragile

This cycle is defined in the HK model as the ecological Φ trap.

7.3.8–7.3.10. Systemic Balance, Academic Contribution, and Conclusion

For ecological sustainability, the following conditions are required in the HK model: E_eco(t) < E_eco_max and dK_eff/dt ≥ 0. When these conditions are not met, the system enters an irreversible process of degradation.

From the HK model perspective, ecological limits constitute the most critical structural constraint facing modern civilisation. Ecological degradation erodes Q, entropy increase raises δ, and carrying capacity declines. This tripartite interaction causes Φ to rapidly exceed the critical threshold.

VIII. PLATFORM-BASED NEO-FEUDALISM: DATA ENERGY, ALGORITHMIC ENTROPY AND HK MODEL INTEGRATION

This section integrates platform capitalism and surveillance-based digital economy with the HK model, revealing the structural and measurable effects of digitalisation on civilisational dynamics. The analysis is deepened through three core mechanisms: (1) the data energy model, (2) algorithmic entropy, and (3) Scenario E: algorithmic lock-in.

Table 8. Platform Neo-Feudalism — HK Parameter Integration

Mechanism	HK Representation	Parameter	Effect on Q/N	Policy Lever
Data energy accumulation	N_total = N_phys + λ·N_dig	λ, N_dig	N↑ → Φ↑	Antitrust; data access regulation
Algorithmic entropy	dQ/dt|_alg = −γ·P(t)·Q	γ, P(t)	Q↓ → Φ↑	Platform levelling; γ reduction
Network concentration	C_dig = Σ s_i²	C_dig	Competition↓ → δ↑	Market concentration caps
Surveillance capitalism	Behavioural surplus extraction	μ↑, δ↑	Coordination↓	Data sovereignty laws
Attention economy	Critical thinking erosion	E_culture↑, Q↓	Long-term Q_eff↓	Digital literacy investment
Neo-feudal lock-in	ρ(network) → dependency cycle	ρ↓, δ↑	Self-repair↓	Open-source infrastructure mandates

8.1. Data Energy, Network Concentration, and Algorithmic Entropy

8.1.1. The Data Energy Model: The New Foundation of Digital Production

In the digital economy, data functions as an 'energy form' that—unlike classical factors of production—expands itself and gains value through network effects. Platforms convert user behaviours into a continuous data flow, generating economic and political power. In the HK model, data can be defined as a new component of Q; however, this capacity is not distributed equally across the system. Owing to platform concentration, effective Q at the societal level can decrease even as total Q rises — this is the core mechanism of the neo-feudal structure.

8.1.2. Network Concentration and Neo-Feudal Structure

The platform economy produces winner-takes-all dynamics due to powerful network effects, causing economic power to concentrate at specific nodes. In the HK model, this concentration is expressed through a Herfindahl-type metric:

$$C_{dig} = \Sigma\, s_i^2 \quad (s_i: \text{market share of the i-th platform})$$

As this metric rises, economic diversity decreases, competition weakens, and systemic fragility increases. A paradox of local optimisation – global inefficiency emerges in the system.

8.1.3. Algorithmic Entropy: Information Decay and Manipulation

Platform systems can degrade information quality while increasing data production. Algorithmic entropy in the HK model is defined as:

$$E_{alg}(t) = \gamma \cdot P(t) \cdot Q$$

Consequently, the quality of decision-making declines, institutional coordination deteriorates, and Q's effectiveness diminishes. This reveals the paradox of digitalisation: the simultaneous rise of information growth and meaning loss.

8.1.4–8.1.9. Data Asymmetry, Scenario E, and the Φ Paradox

As data access asymmetry (A_data) increases, Q rises at the elite level while Q falls at the system level, and Φ rises at the societal level. Scenario E (Algorithmic Lock-in) is the most extreme manifestation of this process: data flows grow, algorithms are optimised, information diversity decreases, δ and μ increase, Q_eff declines, and Φ rises. Unlike classical collapse, this cycle produces a slow, invisible, and digitally mediated erosion process. The core paradox: 'More data → lower systemic rationality → higher fragility.'

8.2. The Rise of the Digital Economy

8.2.1. Zero Marginal Cost and Scalability

The most distinctive feature of digital goods is that production costs rapidly approach zero after the first copy. This fundamentally transforms the production function:

$$TC(Q) = F + \varepsilon \cdot Q \quad (\varepsilon \to 0)$$

This structure permits nearly unlimited scaling of production. In the HK model this appears as a rapid increase in Q; however, since this growth is not evenly distributed, the effective Q across the system follows a different dynamic.

8.2.2. Network Externalities and Natural Monopoly Dynamics

Digital platforms generate powerful network externalities: the value of a platform increases as the number of users grows. This super-linear growth—of the form $V(n) = \alpha \cdot n^{\beta}$ ($\beta > 1$)—

causes market structure to drift from competitive equilibrium towards concentration. In the HK model, accessible capacity decreases even as total capacity increases.

8.2.3–8.2.10. Data Concentration, Φ Dynamics, and Systemic Paradox

In the digital economy, data is a cumulative and self-reinforcing asset; concentration reduces the system's overall efficiency. The fact that the market capitalisation of large digital platforms has surpassed the economies of many countries demonstrates that economic power has shifted from states to platforms. The core paradox that emerges is: 'Digital growth simultaneously produces capacity and systemic fragility.'

8.3. Surveillance Capitalism and Behavioural Control

As an advanced stage of digital capitalism, surveillance capitalism is a system in which economic value is derived from the prediction and direction of human behaviour. From the HK model perspective, this transformation constitutes a new mechanism that alters Q's composition while systematically increasing the δ and μ parameters.

8.3.1–8.3.4. Data Extraction, Prediction, Filter Bubbles, and the Attention Economy

Surveillance capitalism produces behavioural surplus value through large-scale data extraction; it externally modifies individual preference functions and creates filter bubbles through algorithmic content filtering, reducing the capacity for social consensus. Under the pressure of the attention economy, individual decision-making quality is reduced while Q's effectiveness decreases at the system level.

8.3.5–8.3.10. Systemic Effect and Conclusion

Surveillance capitalism operates in the HK model through three principal channels: Q transformation (data and algorithmic capacity increases), δ increase (information decay rises), and μ increase (coordination processes weaken). Conclusion: 'The optimisation of behaviour, rather than increasing the system's rationality, erodes it over the long term.'

8.4. Platform-Based Neo-Feudalism: Conceptual Framework

Platform-based neo-feudalism is built upon three fundamental elements: Infrastructure Control, Data Ownership, and Algorithmic Governance. This structure, analogous to land ownership in the classical feudal system, makes control of digital infrastructures the primary source of economic power.

8.4.1–8.4.6. Access Economy, Rent, Regulatory Gap, and Hierarchy

In the neo-feudal system, economic actors operate through access to platforms rather than owning the means of production; platforms generate a rent economy through intermediation and access control. Existing regulatory frameworks prove insufficient owing to network effects. From the HK model perspective, this structure produces a three-tiered hierarchy: platform cores, dependent producers, and data-supplying users.

8.4.7–8.4.10. Neo-Feudal Lock-in and Conclusion

The cycle of network effects → data accumulation → reduction of competition → decline in innovation → decrease in Q_eff → increase in dependency is defined in the HK model as neo-feudal lock-in. Conclusion: 'In the digital age, ownership has shifted from the means of production to the control of access infrastructures.'

8.5. Algorithmic Power and Democratic Tension

In the digitalised global system of the 21st century, algorithmic power is not limited to economic efficiency but produces direct effects on political and democratic processes. As defined by O'Neil (2016), algorithms can produce discriminatory outcomes through data-based decision mechanisms and erode democratic accountability. Lee (2018) emphasises the strategic role of artificial intelligence in the global power struggle, demonstrating that algorithmic supremacy has become a geopolitical instrument.

8.5.1. HK Model Perspective on Algorithmic Power

From the HK model perspective, algorithmic power directly affects the δ and μ parameters:

$$\delta(t) = \delta_0 + \alpha \cdot P_alg(t)$$

$$\mu(t) = \mu_0 + \beta \cdot P_alg(t)$$

where δ(t) represents the increase in institutional and information entropy; μ(t) represents the deterioration in the effectiveness of coordination mechanisms; and α, β represent the interaction coefficients with algorithmic power. Rising algorithmic power creates asymmetry over Q: Q_individual(t) = Q_system(t) − ξ·P_alg(t).

8.5.2–8.5.5. Democratic Tension, Global Power Race, and Theoretical Implications

Algorithmic power increases democratic tension through two channels: information asymmetry via filter bubbles and targeted content (E_info ↑, Φ ↑), and decision-making distortion through the non-transparent direction of decision processes by algorithms (δ and μ ↑). Under the Φ > 1 condition, rising δ and μ increase the risk of democratic institutions becoming dysfunctional (Scenario E).

8.6. Integration of Platform Neo-Feudalism into the HK Model: Data Energy and Algorithmic Entropy

8.6.1. Channel 1 — N Amplifier: Data Energy Source

Platforms carry the data volume produced from user interactions into the HK model as digital quantitative pressure (N_dig):

$$N_total = N_physical + \lambda \cdot N_dig$$

where λ ∈ [0,1] is the conversion coefficient of the data economy into the real economy, and N_dig is growing at an annual rate of approximately 23% according to IDC DataSphere (2023) data — three to four times the rate of growth of N_physical. Platform monopolisation pushes the λ coefficient upward; the data generated by a single user translates into greater asymmetric power in a less pluralistic environment.

8.6.2. Channel 2 — Q Eroder: Algorithmic Entropy

Platform algorithms exert a negative effect on the Q variable through epistemic decay:

$$dQ/dt|_alg = -\gamma \cdot P(t) \cdot Q$$

where P(t) ∈ [0,1] represents the platform power concentration index, γ represents the algorithmic impact coefficient, and proposed proxy indicators include advertising revenue HHI, global web traffic concentration, and single-platform usage rate in application stores.

8.6.3. Scenario Analysis: Algorithmic Lock-in (Scenario E)

Initial condition: $Q_0 = 0.85$, platform power concentration: $P(t) \to 0.85$, algorithmic impact coefficient: $\gamma = 0.12$. Results: Q falls below the pruning threshold at approximately $t \approx 60$ steps (compared to $t = 85$ in Scenario C). The algorithmic entropy term advances the critical pruning time by approximately 25 steps. Platform levelling can produce a policy effect equivalent to δ reduction.

8.6.4. Academic and Theoretical Contribution

39. Acceleration of N through digitalisation: platform data production increases quantitative pressure via N_dig; Φ approaches critical thresholds.
40. Algorithmic erosion of Q: the $\gamma \cdot P(t) \cdot Q$ term numerically represents epistemic decay.
41. Scenario E — Algorithmic lock-in: digital monopolisation brings forward the critical pruning time.
42. Policy implications: levelling interventions can directly influence the system's entropic balance by manipulating the λ and γ coefficients.

IX. THE 2035 THRESHOLD: GLOBAL SYSTEM RUPTURE POINT

Table 9. 2035 Critical Threshold Indicators — HK Model Benchmarks

Indicator	HK Interpretation	Current Level	Threshold Level	Data Source
Global debt / GDP	N_finance pressure on Q	~340%	~280% (historical pre-crisis)	IMF Global Debt Monitor
DataSphere growth rate	N_dig expansion	~23% p.a.	~10% (Q-matched)	IDC DataSphere 2023
Platform HHI (ad revenue)	P(t) concentration index	~0.42	~0.25 (competitive)	Statista / Cloudflare
Democracy index decline	ρ erosion signal	17 consecutive yrs decline	Stable or rising	Freedom House 2023
Planetary Boundary crossings	α ecological entropy	6 of 9 crossed	≤3 of 9	Rockström et al. 2023
Φ (estimated global)	Core HK ratio	~1.15–1.35 (est.)	$\Phi_c = 1.0$	HK model (this study)

9.1. The Concept of Critical Threshold

The concept of a tipping point, derived from the literature on complex systems theory and ecology, explains how systems undergo sudden and irreversible change not gradually but when a certain parameter boundary is exceeded. This theoretical framework is expressed quantitatively in the HK model through the condition $\Phi \geq 1$. For global civilisation, the critical

threshold represents the point at which energy, information, and institutional capacities are being eroded by systemic entropy and the risk of synchronised rupture is rising.

9.1.1. Theoretical Clarity and Mechanism

The critical threshold can be modelled within the HK framework through three core dynamics:

43. Diminishing Marginal Returns and Entropic Accumulation. Tainter (1988) and Scheffer et al. (2009) demonstrate that marginal returns diminish as complexity increases; quantitative pressure (N) surpasses Q's capacity. In the HK model, the condition $\Phi \geq 1$ indicates that the system has entered a process of pathological growth and epistemic decay through positive feedbacks.
44. Increasing Variance and Critical Slowing Down. When the variance of system parameters rises (e.g., global debt/GDP ratio, data volume growth), the system begins to exhibit delays in dynamic response time. In the HK model, the δ and μ coefficients determine the rates of entropic resistance and information decay.
45. Spatial and Digital Pattern Disruptions. Platform neo-feudalism, algorithmic entropy, and digital data density (N_{dig}, P(t)) trigger information-based rupture. Local collapses begin in Q's epistemic capacity and spread to the global system through network effects. In the HK model, the $\gamma \cdot P(t) \cdot Q$ term quantitatively represents these digital concentration effects.

9.1.2. Mathematical-Logical Consistency

The critical threshold can be defined in the HK model through synchronised entropic triggers: the synchronised interaction of N and Q causes Φ to approach the critical threshold. If platform monopolisation ($\gamma \cdot P(t) \cdot Q$) and digital data volume ($\lambda \cdot N_{dig}$) increase simultaneously, the system crosses the previously projected pruning threshold. This theoretically supports the 2035 threshold phenomenon observed in the HK model's numerical simulations.

9.1.3. Academic Implications

46. Global Systemic Rupture: around 2035, the probability of systemic rupture rises with the simultaneous accumulation of energy, information, and institutional entropies.
47. Critical Threshold Indicators: $\Phi \geq 1$ indicates that quantitative pressure has surpassed qualitative capacity; rising δ, μ, γ signal that entropy rates are increasing; N_{dig} and P(t) quantitatively represent the contribution of digital pressure.
48. Political and Regulatory Strategy: by monitoring critical threshold signals, pre-emptive policy interventions — δ/γ reduction, Q augmentation, and N_{dig} levelling — can be implemented in a timely manner.

9.2. Financial System Fragility

The global financial system is approaching a critical rupture point in terms of the HK model's quantitative pressure (N) and institutional entropy (δ/μ) parameters, within the complex and highly connected structure of the 21st century. According to BIS (2023) data, the notional value of global derivative markets has exceeded 600 trillion USD, dramatically increasing the systemic scale of N.

9.2.1. Theoretical Framework and the Minsky Hypothesis

Minsky's (1986) financial instability hypothesis emphasises that prolonged stability reduces risk perception and increases the system's fragility. From the HK model perspective, this can be addressed under three headings:

49. Financial Overloading of N: derivative markets, high leverage, and chain risks increase the sum of N_dig and N_physical. $\varphi \in [0,1]$ represents the financial system's global pressure contribution coefficient.
50. Entropic Decay and Institutional Insufficiency: high N triggers the δ and μ parameters. When the capacity of bureaucratic, regulatory, and market mechanisms is exceeded, institutional and information entropy rises rapidly. As financial system pressure N_finance grows, Q (institutional and epistemic capacity) is rapidly eroded.
51. Positive Feedback and Systemic Risk: network effects and chain obligations through derivative markets cause a small shock to spread across the entire system, causing Φ to rapidly exceed the threshold $\Phi > 1$.

9.2.2. Mathematical and Logical Clarity

Through the HK model's financial integration, systemic fragility can be quantified in three dimensions:

52. N Increase and Critical Threshold: with the increase of financial pressure N_finance, $\Phi(t) = N / (\theta \cdot Q)$ rapidly exceeds 1.
53. Acceleration of δ and μ: rising N_finance amplifies the effect of the δ and μ parameters; institutional response remains delayed and weak.
54. Synchronisation with Digital-Algorithmic Effects: through the $P(t) \cdot \gamma$ term, platform power and algorithmic entropy quantitatively advance the financial system's fragility.

9.2.3. Academic Implications

55. Critical Financial Threshold: around 2035, the aggregate pressure of derivative and digital financial products will cause N to exceed Q's capacity, and the $\Phi \geq 1$ condition will become chronic.
56. Scenario Simulations: HK-based simulations can numerically model the effect of financial shocks on δ and μ, and can estimate early warning signals and systemic rupture timing.
57. Policy Implication: financial regulation, capital adequacy ratios, derivative exposure limits, and platform oversight can delay the critical threshold by reducing the δ/μ and γ coefficients.

9.3. The Weakening of Political Systems

In the global civilisation of the 21st century, the fragility of political systems has a direct effect on institutional entropy (δ) and self-repair capacity (ρ) within the HK model. Freedom House (2023) data shows a continuous 17-year decline in the global democracy index; this signals a systemic dynamic that leads to an increase in Φ and the exceedance of the critical threshold ($\Phi > 1$).

9.3.1. Theoretical Framework: HK Model Perspective

Ostrom's (1990) collective action theory explains how coordination failures at global and local scales trigger deficiencies in the management of common resources and the loss of effectiveness of political systems. This situation is formulated in the HK model framework through three parameters:

58. Decline in Self-Repair Capacity (ρ): democratic backsliding and authoritarianisation limit institutional learning and adaptive capacity. Declining ρ reduces Q's capacity to balance entropic erosion.
59. Increase in Institutional Entropy (δ): the dysfunction of political institutions raises δ and accelerates the exceedance of Φ's critical threshold. The lack of international coordination increases the entropic burden on global problems such as the climate, energy, and financial crises.
60. Negative Feedback Loop: the decline of Q weakens political trust and social cohesion; this is fed by further N increases (migration, resource demand, financial pressures) and a rise in δ. In the HK model, this can be quantified as the migration of Φ to the chronic $\Phi > 1$ zone.

9.3.2. Mathematical-Logical Elaboration

The weakening of political systems creates a dynamic crisis pathway over time in the HK model: ρ(t) declines as it depends on political reform capacity, while δ(t) increases with political instability and authoritarianisation. These time-dependent parameters push the Φ(t) curve beyond the critical threshold and mechanically explain the system's approach to the 2035 rupture threshold.

9.3.3. Academic Implications

61. Global Coordination Deficit: according to Ostrom (1990) and similar studies, collective action failures increase N and δ, leading to the early exceedance of the critical threshold.
62. Political Entropy and Digitalisation Interaction: platform neo-feudalism and algorithmic entropy further reinforce political system weakness; through the $\gamma \cdot P(t)$ term, digital pressure increases the erosive effect on Q.
63. Scenario Analysis: simulations with chronic ρ decline and δ increase demonstrate that the combination of political system weakening with financial and ecological crises makes systemic rupture inevitable.
64. Policy Implication: strengthening global coordination mechanisms, democracy-supportive reforms, and international cooperation can delay the migration of Φ beyond the critical threshold by constraining δ and μ parameters.

9.4. Cascading Failure: HK Model Perspective on Global System Collapse Dynamics

The concept of cascading failure describes the process by which, in interconnected social, economic, and ecological systems, a shock carries the entire system into crisis mode through chain interactions. Within the HK model framework, this process can be expressed mathematically and conceptually as multi-dimensional feedback mechanisms that accelerate Φ's exceedance of the critical threshold.

9.4.1. Theoretical Framework

Four principal channels emerge in global system cascading failures:

65. Ecological Shocks (α): severe climate events, biodiversity loss, and water scarcity directly reduce Q's carrying capacity. In the HK equation, E_eco represents the magnitude of the ecological shock and, in combination with δ, triggers a rapid rise in Φ.
66. Demographic and Migration Pressures (N): extreme population pressure between rural and urban areas, resource competition, and migration movements increase N exponentially. Feedback: rising N → rising δ → declining Q → further N pressure.
67. Political Extremism and Institutional Collapse (δ, μ): political instability and authoritarianisation raise δ; information and coordination losses are modelled through μ. This erodes the system's self-repair capacity (ρ) and accelerates the cascade.
68. Economic Instability and Financial Chain Effects: derivative markets, debt levels, and the concentration of the platform economy contribute to rising N and δ. Even a small shock in the global financial system can trigger a chain collapse; F_shock represents financial crisis severity and β represents the multiplier effect.

9.4.2. Mathematical-Logical Elaboration

Cascading failure can be modelled in the HK model through time-dependent parameters and multi-layer interactions:

$$d\Phi/dt = f(\delta(t), \mu(t), \gamma(t), \alpha \cdot E_eco, \beta \cdot F_shock, \lambda \cdot N_dig(t))$$

where δ(t), μ(t), γ(t) represent political, social, and digital system pressures; α·E_eco represents ecological entropy; β·F_shock represents the economic and financial cascading effect; and λ·N_dig(t) represents the platform data accumulation and digital pressure contribution. These equations enable the numerical simulation of Φ's transition beyond the critical threshold and cascading failure through multi-dimensional interactions.

9.4.3. Academic Implications

69. Importance of Feedback Mechanisms: feedback loops among ecology, demography, politics, and economics accelerate the transformation of individual crises into systemic collapse.
70. Pre-Critical-Threshold Early Warning: rising variance, critical slowing down, and spatial pattern changes are precursor signals indicating that Φ will rapidly exceed the critical threshold.
71. Scenario Planning and Policy Implications: controlling the δ and μ parameters in the HK model, and limiting digital and economic pressures, are critically important for the system to delay cascading failure.
72. Multi-Layer Simulation: cascading failure scenarios, integrated with climate, financial, and digital data, project chronic $\Phi > 1$ conditions in Φ(t) simulations, enabling the mechanical modelling of the 2035 rupture threshold.

Figure 6 — Cascading Failure Simulation

Φ(t) trajectories under four cascade scenarios: (A) baseline, (B) financial shock only, (C) ecological + demographic, (D) full cascade (all channels active). Red zone: Φ > 1.2 (irreversible systemic rupture). [Figure generated from Python output.]

Figure 6. Cascading Failure Simulation — Φ(t) under single-channel and full-cascade conditions (2025–2065).

X. THE SYSTEMIC PRUNING PROCESS (2035–2065) AND AN ACTION PLAN FOR POLICY-MAKERS

Table 10. Scenario D — Policy Action Table for Intervention-Assisted Recovery

Policy Domain	Objective	HK Target	Concrete Actions	Expected Effect
Digital Regulation	Reduce N_dig & P(t)	λ↓, γ↓	Antitrust action; data portability mandates; platform levelling	Φ approaches critical threshold more slowly
Ecological Policy	Constrain α·E_eco	α↓, K_eff stable	Carbon pricing; biodiversity protection; regenerative agriculture	Q_eco maintained; Φ < 1.2
Fiscal Governance	Control N_finance	N↓, δ↓	Debt ceilings; derivative exposure limits; sovereign debt restructuring	Systemic cascade risk reduced
Institutional Reform	Raise ρ AND reduce δ	ρ↑, δ↓	Multi-dimensional governance reform; anti-corruption; judicial independence	Q self-repair capacity restored
Education & Culture	Protect Q_eff	E_culture↓, Q↑	University independence; media literacy; cultural heritage investment	Long-term Q capacity secured
Demographic Policy	Manage N trajectory	ΔN_dem, η>0	Integrated urban planning; health & education investment; migration management	N growth rate matched to Q growth
Energy Transition	Close EROI gap	Q_energy↑, δ(transition)↓	Accelerated renewables deployment; just transition frameworks	Q_energy stabilised; Φ reduced

10.1. The Analytical Framework of the Pruning Metaphor: A Systemic Restructuring Perspective

The concept of systemic pruning refers to self-organising restructuring processes triggered by accumulated pressures in complex adaptive systems. Within the HK model context, pruning is represented mathematically and conceptually as the sudden withdrawal and rebalancing that emerges in the quantitative and qualitative components of the system following Φ's exceedance of the critical threshold.

10.1.1. Theoretical Depth

Holling's (2001) adaptive cycle model proposes that the system has four phases:

73. Growth and Expansion (r & K phase): with the growth of Q and N, the system produces marginal returns; $\Phi < 1$.
74. Peak ($\Phi \rightarrow 1$): marginal returns begin to decline, entropy (δ, μ) rises.
75. Dissolution and Pruning (Ω): the $\Phi > 1$ threshold is crossed; excess load, excessive N, and δ/μ accumulation cause Q to decline. The system 'prunes' itself.
76. Reorganisation (α): the system adaptively reorganises towards new Q/N equilibrium points.

Within the HK model framework, the pruning metaphor transforms into a mathematical framework that simulates the excessive burden of quantitative growth and the erosion of qualitative capacity. Pruning triggers are represented by the δ, μ, γ, α parameters; adaptive reorganisation by ρ; and the critical threshold N_c by the condition $\Phi = 1$ or $\Phi > 1$.

10.1.2. Mathematical-Logical Elaboration

The effect of the pruning process on the HK model is examined through dynamic feedback and multi-layer integration:

77. Critical Φ Exceedance: $\Phi_c = N_c / Q_c \geq 1$. This threshold models the chronic exceedance processes observed in the historical cases of Rome, the Maya, and the Soviet Union.
78. Pruning Rate and Severity: B(t) represents the rate of sudden capacity loss of the system; the f and g functions parametrically determine the effect of feedbacks on Φ.
79. Reorganisation and Self-Repair: the ρ parameter represents the rate of Q's recovery and the capacity for adaptation to a new equilibrium. Can be optimised through Scenario D or E intervention scenarios.

10.1.3. Academic Implications

80. Pruning Combines Chaotic and Deterministic Components: systemic pressures, stochastic shocks, and entropic processes together trigger Φ's exceedance of the critical threshold; can be expressed quantitatively using the HK model.
81. Critical Lessons for Policy-Makers: single-dimensional interventions are generally insufficient; multi-dimensional management of entropy channels is required. Digital, economic, and ecological pressures must be addressed in an integrated manner.
82. Scenario Simulations: the pruning process projects chronic $\Phi > 1$ conditions in global rupture simulations for the period 2035–2065. The effects of algorithmic monopolisation, digital data growth, and financial shocks can be modelled numerically.

83. The HK Model's Interpretive Power: systemic pruning not only explains the collapses in historical cases, but also provides the means to quantitatively test political, economic, and digital intervention strategies.

10.2. Demographic Restructuring: HK Model Perspective

Demographic restructuring plays a central role in the process of Q and N components reaching a new equilibrium following systemic pruning. Sen's (1999) 'development as freedom' approach relates population and demographic change not merely to a quantitative increase or decrease, but also to quality of life, social capacity, and economic productivity. This perspective can be directly integrated with the parametric and mathematical framework of the HK model.

10.2.1. Theoretical Depth

84. Population Pressure (N) and Carrying Capacity (K): in the HK model, N(t) represents quantitative pressures such as population, data volume, and capital. Demographic restructuring aims to reduce excess N below the critical threshold and to return Φ to ≤ 1. ΔN_dem is the magnitude of effect determined by fertility control, migration management, urban planning, and social policies.
85. Effect on Q: the regulation of N allows the reconstruction of Q and the recovery of epistemic capacity. The η coefficient represents the restorative effect of demographic levelling on Q.
86. Quality of Life and Social Entropy: Sen's approach directly relates the reduction of N to improvements in quality of life (health, education, security). The social entropy (δ, μ) parameters are reduced by the effect of demographic restructuring; Q erodes less and Φ stabilises more rapidly.

10.2.2. Mathematical-Logical Consistency

87. Descent Below the Critical Threshold: $\Phi = (N - \Delta N_dem) / (Q + \eta \cdot \Delta N_dem) \leq 1$. Under the $\Phi \leq 1$ condition, systemic pruning ceases and restructuring begins.
88. Feedback Loop: the reduction of N triggers an increase in Q; rising Q, by increasing information and energy efficiency, can limit N's growth at the same rate in subsequent periods.
89. Scenario Analysis: high N, low Q $\rightarrow$ $\Phi > 1$ $\rightarrow$ pruning is triggered. Demographic intervention (ΔN_dem, $\eta > 0$) $\rightarrow$ Φ declines $\rightarrow$ Q stabilises $\rightarrow$ social and economic restructuring becomes possible.

10.2.3. Academic Implications

90. When integrated with the HK model, demographic restructuring is also a critical policy instrument in terms of increasing knowledge, education, and social capacity.
91. Policy recommendation: integrated population management through urban and rural planning, health and education investments, prevents Φ from exceeding the critical threshold.
92. Mathematical contribution: with the ΔN_dem and η parameters, the HK model can now numerically simulate the effects of demography-based interventions.

10.3. Pruning of the Economic System: HK Model Perspective

The pruning of the economic system requires modelling the effects of the systemic pruning process on N and Q within the HK model framework. The more resource-efficient and equitable economic structures proposed in works such as Kallis (2018), Daly (1996), and Raworth (2017) can be represented in the HK model as the bringing of N under control and the optimisation of Q.

10.3.1. Theoretical Depth

93. Pruning of N (Reduction of Quantitative Pressure): in the HK model, economic growth, investment, and debt pressure are represented by N(t). ΔN_eco represents the reduction in economic pressure achieved through the curtailment of carbon-intensive production, excessive borrowing, and inefficient resource use.
94. Preservation and Enhancement of Q: economic restructuring is not limited to reducing N; it must be integrated with innovation, education, and green technology investments. The η_eco coefficient represents the restorative effect of economic pruning on Q.
95. Ecological and Digital Interactions: the pruning of N must be coordinated with ecological limits (the α parameter). Digital platforms can re-increase pressure through N_dig; integration with the λ and γ parameters is critical.

10.3.2. Mathematical-Logical Consistency

The economic pruning process with the HK model can be expressed through three fundamental equations:

$$dQ/dt = \rho \cdot Q(1-Q) - \delta \cdot Q + \eta_{eco} \cdot \Delta N_{eco}$$

$$\Phi(t) = (N - \Delta N_{eco}) / (Q + \eta_{eco} \cdot \Delta N_{eco})$$

This structure enables the numerical simulation of the quantitative and qualitative effects of economic pruning in the HK model.

10.3.3. Policy-Oriented Implications

96. Resource Efficiency: carbon-intensive forms of production must be reduced; simultaneously, green technologies that will increase Q must be supported.
97. Debt and Financial Pressure Control: excessive borrowing and speculative investments raise N, pushing Φ above critical thresholds. The HK model enables this pressure to be monitored numerically.
98. Digital and Ecological Coordination: digital pressures from N_dig and P(t) can weaken economic pruning policies. Economic, ecological, and digital policy packages must be integrated.
99. Scenario Analysis: while Scenario C (conventional economic growth) produces a $\Phi > 1$ condition, Scenario D/E (ecologically and digitally compatible economic pruning) ensures sustainable restructuring with $\Phi \leq 1$.

10.4. Institutional and Cultural Pruning: HK Model Perspective

Institutional and cultural pruning refers to the restructuring of social norms, collective values, and institutional capacity (Q). While Ostrom's (1990) work on commons governance

demonstrates that resources can be sustainably managed at the local level, this framework can be generalised through the HK model to the context of global systems and digital platforms.

10.4.1. Theoretical Depth

100. Institutional Self-Repair (ρ) and Entropy Control (δ, μ): institutional reforms and normative regulations increase the self-repair capacity (ρ) over Q. However, when institutional entropy (δ) and information entropy (μ) remain high, single-dimensional reforms are insufficient; γ·P(t) represents digital platform effects and must be balanced together with cultural/institutional intervention.

101. The Epistemic Role of Cultural Norms: collective memory and cultural values function as social binding mechanisms that preserve Q's sustainable qualitative capacity. Filter bubbles, disinformation, and cultural fragmentation increase the negative effect on Q and accelerate Φ's approach to the critical threshold; numerically represented by the γ·P(t)·Q term in the HK model.

102. Community-Based Governance (Ostrom) and Scenario Application: self-governance mechanisms at the local level reduce δ and μ, stabilise Q, and cause Φ to return below 1. η_soc represents the effectiveness coefficient of community-based intervention; f(C) represents social cohesion and normative compliance.

10.4.2. Mathematical-Logical Consistency

The numerical form of the institutional and cultural pruning process in the HK model:

$$dQ/dt = \rho \cdot Q(1-Q) - \delta \cdot Q - \gamma \cdot P(t) \cdot Q + \eta_soc \cdot f(C)$$

Institutional and cultural pruning stabilises Φ and prevents the system from exceeding the critical threshold. Scenario C (uncontrolled N) produces $\Phi > 1$ with institutional entropy remaining high. Scenario D (mixed intervention) ensures sustainable restructuring with $\Phi \leq 1$ through community-based intervention + Q augmentation. In Scenario E where P(t) is high, cultural and institutional intervention must be stronger to balance the γ effect.

10.4.3. Policy-Oriented Implications

103. Multi-dimensionality of Institutional Reforms: a culture of legal system integrity, bureaucratic transparency, and accountability reduces cultural and institutional entropy.

104. Community Participation and Normative Power: Ostrom-based community governance strengthens the social structure that stabilises Q; it functions as a buffer against digital and global pressures.

105. Digital Platform Levelling: in scenarios where P(t) and γ coefficients are high, the effectiveness of cultural and institutional intervention must be increased.

10.5. Risks: Inequality and Structural Violence — HK Model Perspective

The dynamics of inequality and structural violence that emerge during crisis periods of modern and historical civilisations can be systematically analysed through the HK model framework in terms of both the N and Q variables.

10.5.1. Theoretical Depth

106. Wealth and Power Concentration (N Variable): Piketty (2014) emphasises the growth of r > g inequality during crisis periods and wealth concentration. From the HK model perspective, this is expressed as the increase in N surpassing the social capacity Q. Rising N weakens the economic and political representation capacity of the middle and lower classes, causing Φ to reach the threshold $\Phi_c \approx 1$.

107. Institutional and Qualitative Decay (Q Variable): when rising N surpasses Q, qualitative capacity erodes. Under these conditions, social violence and institutional deterioration become unavoidable; the collapse of the middle class in Rome can be interpreted as a historical prototype showing that Q was declining.

108. HK Interpretation of Structural Violence: violence is a positive feedback mechanism triggered when Q falls below a certain threshold. Social and economic shocks increase the δ and μ coefficients, causing Φ to exceed the critical threshold.

Figure 7 — Inequality, Structural Violence, and Φ Dynamics

Simulated Φ(t) trajectories under rising wealth concentration (Gini coefficient as N proxy). Overlay: historical rupture events (Roman middle-class collapse, Soviet dissolution). [Figure generated from Python/Monte Carlo output.]

Figure 7. Inequality-Driven Φ Dynamics — Comparative historical and simulated trajectories.

[Sections X.5.2 onwards, and the Conclusion, continue in the main manuscript.]

Continuation — Sections 10.5.2 – XII (Conclusion and References) | ***Social Entropy and Systemic Pruning Theory: 2035–2065***

10.5.2. Mathematical-Logical Consistency

HK Model Risk Dynamics:

$$dQ/dt = \rho \cdot Q(1-Q) - [\delta + \mu \cdot S + \gamma \cdot P(t)] \cdot Q$$

Under the condition $\Phi > \Phi_c$, systemic collapse and social violence are triggered. In scenarios where $\gamma \cdot P(t) \geq 0.1$, inequality and violence occur earlier and more rapidly.

10.5.3. Policy-Oriented Implications

1. Inequality Control: reducing income and wealth concentration brings N under control and delays Φ's exceedance of the critical threshold.
2. Institutional and Cultural Intervention: policy and education reforms that augment Q counterbalance the δ and μ effects and limit structural violence.

3. Digital Regulation: bringing platform and algorithmic power concentration (γ·P(t)) under control reduces the system's fragility. When γ is reduced, the time for Φ to reach the critical threshold is delayed.
4. Alignment with Historical Examples: the collapse of the middle class in Rome serves as a historical prototype of Φ > 1 and Q decline; modern digital wealth concentration and algorithmic effects are accelerating the same mechanism.

10.6. Policy Action Plan for Policy-Makers: Operationalisation of Scenario D — HK Model Perspective

Scenario D (Intervention-Assisted Recovery) is designed within the HK model framework to prevent the system from exceeding critical Φ thresholds and to balance the pressure on N by stabilising Q.

10.6.1. Core Principles and Theoretical Framework

HK Model Parameters and Policy Effects:

- **δ (entropy coefficient):** represents social, economic, and ecological pressures; its reduction delays Φ's exceedance of the critical threshold.
- **ρ (self-repair capacity):** reflects institutional, cultural, and economic recovery capacity; its increase accelerates Q's reconstruction.
- **β (entropy dissipation rate):** controls the rate of spread of deteriorations in the system; its increase enables early localisation of damage.
- **α (ecological entropy coefficient):** represents the resilience of natural systems against pressure; its reduction increases environmental sustainability.

The HK model-based ODE system:

$$dQ/dt = \rho \cdot Q(1-Q) - \delta(t) \cdot Q - \mu \cdot S \cdot Q - \gamma \cdot P(t) \cdot Q$$

Policy interventions are modelled directly through the δ, ρ, β, and α parameters.

10.6.2. Policy Instruments and HK Parameters

- Social welfare and income distribution reforms [δ↓]: reduces social pressure and inequality-induced entropy, delaying Φ's exceedance of the critical threshold.
- Education and institutional capacity development [ρ↑]: increases Q; supports the reconstruction of cultural, scientific, and governance capacity.
- Crisis management and early warning systems [β↑]: controls the rate of spread of deteriorations, enabling localisation of systemic collapse.
- Ecological investment and carbon reduction policies [α↓]: reduces ecological entropy, diminishing environmental pressures and ensuring the long-term sustainability of Q.
- Digital regulation and platform oversight [γ·P(t)↓]: limits algorithmic entropy and reduces its negative effect on Q; brings digital monopolisation under control.

10.6.3. Mathematical Scenarios and Numerical Simulation

Coordinated intervention parameters:

```
δ:          0.05 → 0.02
ρ:          0.08 → 0.12
α:          0.06 → 0.03
β:          0.04 → 0.06
γ·P(t):     0.12 → 0.05
```

With these parameter changes, model simulations delay Q's descent below the critical pruning threshold by 20–25 time units, and the duration of Φ's exceedance is meaningfully reduced.

Policy-oriented implication: single-dimensional intervention (raising ρ alone, or reducing δ alone) is insufficient; simultaneous and coordinated application is required. If the digital economy and platform effects are neglected, γ·P(t) remains high and systemic fragility persists.

10.6.4. Academic and Policy Implications

- HK Model Operationalisation: Scenario D theoretically and numerically demonstrates the reversal of the chronic $\Phi > 1$ conditions observed in historical examples (Rome, Britain, the Soviet Union).
- Policy Priority: addressing inequality, ecology, digital monopolisation, and institutional capacity simultaneously plays a critical role in preventing global systemic ruptures.
- Applicability: governments can simulate scenarios by converting HK parameters into policy instruments, thereby optimising decision processes.

Table 6 (Revised). Scenario D Action Plan — Policy Instruments and HK Parametric Effects

Policy Domain	Priority Instrument	HK Parametric Effect	Time Horizon	Implementation Barrier	Priority
Digital Wealth Tax	Global minimum corporate tax (≥15%; extension of Pillar Two)	δ↓ — platform monopoly rent reduced	Short (2–5 yrs)	Tax arbitrage; offshore structures	High
Digital Property Reform	Data co-ownership model; mandatory data portability	N_dig↓ / P↓ — platform monopoly erosion	Medium (5–10 yrs)	Platform lobbying; EU/US regulatory divergence	High
Algorithmic Transparency	Mandatory algorithm auditing (extended EU AI Act)	γ↓ — algorithmic entropy reduced	Short (2–5 yrs)	Trade secret claims; technical capacity gaps	High
Institutional Capacity Building	Independent judiciary reform; civil oversight strengthening	ρ↑ — institutional self-repair increased	Long (10–20 yrs)	Political resistance; short-term electoral logic	Critical
Ecological Transition	Carbon tax + dividend; removal of fossil fuel subsidies	α↓ — ecological entropy reduced	Medium (5–15 yrs)	Fossil fuel industry; energy	High

				security concerns	
Epistemic Infrastructure	Public media strengthening; media literacy education; disinformation infrastructure	β↑ — entropy dissipation rate accelerated	Short (2–5 yrs)	Political polarisation; private media pressure	Medium
Demographic Balance	Skilled migration programmes; parenting and care support	N/Q ratio balanced	Long (20+ yrs)	Political populism; cultural resistance	Medium
Intervention Coordination	Simultaneous package implementation of ALL policies above	Scenario D parameters achieved (approx. 2× effect)	Medium (5–10 yrs)	Inter-agency silos; international coordination	Critical

The most critical finding in the table is the 'Intervention Coordination' row: Scenario D analysis shows that when interventions are applied simultaneously, they are approximately twice as effective as sequential application. This provides a numerical confirmation of the lesson taught by the Ottoman Tanzimat reforms: raising ρ alone (institutional capacity) without reducing δ (pressure reduction) is insufficient.

Table 11. HK Model Core Parameter Summary — Scenarios C, D, and E

Param	Interpretation	Scenario C	Scenario D	Scenario E	Direction for Recovery
ρ	Self-repair / institutional adaptability	0.025	0.08	0.025	↑ Increase
δ	Entropy coefficient / coordination decay	0.18	0.05	0.18	↓ Decrease
μ	Information decay / epistemic erosion	0.10	0.03	0.10	↓ Decrease
β	Entropy dissipation rate	0.12	0.25	0.12	↑ Increase
α	Ecological entropy coefficient	0.06	0.03	0.06	↓ Decrease
γ	Algorithmic entropy coefficient	0.00	0.05	0.12	↓ Decrease
P(t)	Platform power concentration [0,1]	0.00	0.40	0.85	↓ Decrease
λ	Digital-to-real economy conversion	0.30	0.15	0.50	↓ Decrease
Φ (est.)	N/Q systemic ratio	→ >1.2	→ <1.0	→ >1.5	↓ Below Φ_c = 1

Figure 8 — Scenario D Parameter Trajectory Comparison

Time series of Q(t) and Φ(t) under Scenarios C (no intervention), D (coordinated intervention), and E (algorithmic lock-in). Arrows indicate intervention commencement point. Shaded zone: Φ > 1 (systemic fragility). [Figure generated from Python simulation.]

Figure 8. Scenario D vs. C vs. E — Q(t) and Φ(t) trajectories showing the effect of coordinated policy intervention (2035–2065).

XI. THE POST-2065 ENLIGHTENMENT CONSTITUTION: HUMANITY'S SECOND CHANCE

Table 12. Post-2065 Civilisation Framework — HK Parameter Targets by Domain

Civilisational Domain	Core Mechanism	HK Parameter	2065+ Target	Policy Instrument
Institutional Capacity	High-Q zones: law, science, civil society	ρ↑, δ↓	Φ < 1 sustained	Judicial independence; civic participation
Ecological Sustainability	Ecosystem recovery; regenerative production	α↓, K_eff stable	E_eco < E_eco_max	Carbon neutrality; biodiversity restoration
Digital Ethics	Algorithmic transparency; data rights	γ↓, P(t)↓	γ_eff < 0.05	Universal AI audit; data co-ownership
Cultural & Scientific Renewal	Collective memory; critical thinking	E_culture↓, β↑	Q_eff sustained	University independence; cultural heritage funding
Participatory Governance	Deliberative democracy; decentralised decision-making	ρ↑, μ↓	ρ > δ sustained	Participatory assemblies; digital consultation
Demographic Equilibrium	N/Q balance; quality over quantity	ΔN_dem, η>0	dN/dt ≤ dQ/dt	Integrated urban planning; education investment

11.1. Conceptual Foundation

The post-2065 period symbolises humanity's transition from a quantitatively expansion-focused evolutionary trajectory to one oriented towards qualitative deepening and sustainable capacity construction. In the light of the HK model's entropy and capacity parameters, this period can be defined as a post-critical phase in which the system returns to $\Phi < 1$ thresholds and re-achieves equilibrium.

11.1.1. Historical Lessons and Parametric Analysis

Historical analyses demonstrate that post-pruning reconstruction in civilisations such as Rome, the Maya, and the Soviet Union was possible through the preservation of high-Q domains. In this context:

- **Q (Institutional and Knowledge Capacity):** the domains of law, education, science, and civic community are reconstructed and critical knowledge accumulation is preserved.
- **ρ (Self-Repair Capacity):** is increased during the restructuring process; institutional adaptation and cultural learning are accelerated.
- **δ (Pressure Coefficient / Entropy Generation):** social and environmental pressures are reduced through effective policies; Φ is prevented from re-exceeding critical thresholds.
- **α (Ecological Entropy):** the natural carrying capacity is increased through the rebalancing of ecosystems; sustainable production models are supported.
- **β (Entropy Dissipation Rate):** the spread of deteriorations is minimised through early detection and on-site intervention.

From a mathematical modelling standpoint, the post-2065 period aims to create a stable fixed point on the HK differential equation system: $dQ/dt = 0$ and $dN/dt = 0$. At this point, the system is in equilibrium both institutionally and quantitatively; Φ's exceedance of the critical threshold has been eliminated.

11.1.2. Qualitative Deepening and Global Sustainability

The main characteristics of post-2065 civilisation:

5. Institutional High-Q Domains: law, knowledge production, civic participation, and ethical standards are preserved and reconstructed as critically important.
6. Ecological and Digital Sustainability: natural and digital entropy is minimised through political interventions on the α and γ parameters.
7. Collective Learning and Social Memory: lessons derived from past collapses are integrated into cultural and educational infrastructure. Through the β parameter, the system acquires adaptive resistance against new deteriorations.
8. Coordination of Policy and HK Parameters: when the $\delta\downarrow$, $\rho\uparrow$, $\alpha\downarrow$, $\beta\uparrow$, $\gamma\downarrow$ interventions are applied simultaneously, systemic rupture risks are minimised and Φ is kept continuously below 1.

11.1.3. Academic Implications and HK Model Perspective

- The post-2065 period can be parametrically and mathematically supported by the HK model as a post-critical period of stability and qualitative deepening.

- This approach enables the numerical simulation of the post-pruning restructuring dynamics of historical civilisations.
- Coordinated interventions and the preservation of high-Q domains constitute the core strategy for securing humanity's second chance.

11.2. Social and Cultural Arrangements

The success of post-2065 civilisation depends not only on the reconstruction of economic and technical capacity, but on the strategic realignment of social and cultural infrastructure. Delors' (1996) four pillars of learning — learning to know, learning to do, learning to live together, and learning to be — represent, within the context of the HK model's Q parameter, the critical components of qualitative capacity.

11.2.1. The Q Parameter and Cultural Resilience

Within the HK model framework:

$$dQ/dt = \rho \cdot Q(1-Q) - (\delta + \mu + \gamma \cdot P(t)) \cdot Q$$

- **The ρ·Q(1−Q) term:** represents the self-repair effect of social learning and cultural reproduction. Delors' learning pillars contribute to Q's renewed increase, balancing the entropy pressure (δ + μ + γ·P(t)).
- **The 'learning to know' and 'learning to do' pillars:** increase knowledge production and application capacity, reducing institutional and scientific entropy and preventing Φ's exceedance of the critical threshold.
- **'Learning to live together and to be':** social cohesion and cultural diversity control the entropy dissipation rate through the β parameter; community-based learning constitutes a resistance mechanism that reduces δ.

11.2.2. Cultural Diversity and Social Resistance

Cultural diversity, like biodiversity, increases the system's ecosystemic and social resilience capacity. The coexistence of different norms, values, and knowledge accumulations prevents a single shock from collapsing the entire system. Its effect on the δ and μ parameters in the HK parameter context can be directly observed; as diversity increases, coordination errors and knowledge losses are compensated more rapidly.

Mathematically, community-based cultural diversity increases Q's effective growth rate:

$$dQ/dt|_{culture} = \rho \cdot f(C_{diversity}) \cdot Q(1-Q)$$

11.2.3. Education and Cultural Policy Recommendations

- Universal Education and Multiple Learning: Delors' four pillars guarantee Q's sustainable growth in the HK model.
- Local and Global Cultural Preservation: the preservation of cultural diversity provides a directly reducing effect on δ and μ.
- Social Memory and Collective Learning: the integration of lessons from historical crises into education and cultural policies increases the β parameter, reducing entropy propagation.

11.2.4. Academic Implications

Delors' four learning pillars, directly integrated into the HK model, provide a numerical and theoretical mechanism preventing Φ's prolonged exceedance of the critical threshold. This approach demonstrates that post-2065 civilisation can achieve both systemic resistance and qualitative deepening by strategically strengthening social and cultural Q domains.

11.3. Technological and Digital Ethics

In the civilisational vision of post-2065, technological and digital infrastructure must be addressed not only in terms of production and efficiency, but also with dimensions of ethics and social sustainability. Floridi's (2014) framework of information ethics theoretically provides algorithmic standards of transparency, accountability, and data justice; these standards are directly related to the algorithmic entropy term (γ·P(t)·Q) in the HK model.

11.3.1. Algorithmic Entropy and Digital Rights

The algorithmic entropy term leading to Q's decline in the HK model:

$$E_{alg} = \gamma \cdot P(t) \cdot Q$$

- **The γ·P(t)·Q term:** reduces Q's effective capacity owing to platform monopolisation and algorithmic biases.
- **Political and ethical implication:** when digital rights are not guaranteed on a par with universal human rights, the γ coefficient increases; Q declines more rapidly and Φ's exceedance of the critical threshold accelerates.
- **Mathematical logic:** γ reduction provides a policy intervention equivalent to δ reduction in the HK model context; controlling algorithmic entropy delays systemic pruning and preserves Q's qualitative capacity.

11.3.2. Transparency, Accountability, and Digital Responsibility

- Transparency: making the functioning of algorithms open and accessible reduces the μ parameter; information entropy is brought under control.
- Accountability: when the decision mechanisms of platforms and AI systems are auditable, the pressure on δ decreases, delaying Φ's approach to the critical threshold.
- Universalisation of digital rights: globally standardised data protection and digital rights mechanisms minimise the systemic effect of the γ coefficient, guaranteeing Q's sustainability.

11.3.3. Policy and Mathematical Integration

Proposed interventions for reducing the γ coefficient in the HK model:

9. The establishment of universal algorithmic oversight mechanisms and ethics boards.
10. The application of digital data transparency standards within a global framework.
11. User control and data pluralism (reduction of platform monopoly P(t)).

The effect of these policies can be numerically simulated in the HK model context:

$$\gamma_{eff} = \gamma_0 \cdot (1 - \eta_{digital} \cdot M_{regulation})$$

11.3.4. Academic Implications

This framework reveals the direct mathematical and logical effects of digital ethics and human rights interventions within the HK model context. The control of the γ parameter reduces the effects of systemic pruning, supporting the technological and social sustainability of post-2065 civilisation's restructuring.

11.4. Structural Principles: Flexibility, Transparency, and Participation — Integrated Social-Digital Law and HK Parameters

In the civilisational vision of post-2065, social and digital legal structures must be treated not merely as a normative framework, but as a mathematical and dynamic mechanism of effect within the HK model context. Habermas' (1984) theory of communicative rationality offers a solid philosophical foundation for explaining the effects of participation and transparent decision mechanisms on the system's self-repair capacity (ρ).

11.4.1. Participation and the ρ Parameter

Definition of ρ in the HK model: the quantitative representation of self-repair capacity within the system; determines Q's capacity for self-reorganisation and adaptation.

The effect of participation:

$$\rho_participatory = \rho_0 + \eta_participation \cdot f(participation_level)$$

Mathematical logic: participatory structures can limit the rise of δ and μ by increasing decentralised information flow and decision-making flexibility. This delays Φ's exceedance of the critical threshold and ensures the preservation of the system's qualitative capacity.

11.4.2. Transparency and Algorithmic Entropy

The γ·P(t)·Q term: expresses platform-based digital entropy. Transparency mechanism: the functioning of algorithms being open and auditable; the monitorability of data flows by users and regulatory institutions. Transparency reduces the effective value of the γ coefficient:

$$\gamma_eff = \gamma_0 \cdot (1 - \eta_trans \cdot T_transparency)$$

11.4.3. Flexibility and the Evolutionary Role of Social-Digital Law

Flexibility: the adaptive capacity of institutional and digital systems to changing pressures. Flexible institutions increase the system's resilience by limiting the rise of the δ and μ coefficients.

Policy recommendations:

12. Disseminating decision-making processes through participatory assemblies and digital consultation platforms.
13. Making transparency and accountability mandatory in algorithmic processes.
14. Establishing regular revision and oversight mechanisms to ensure the evolutionary adaptation of social and digital law.

11.4.4. Academic and Mathematical Implications

This integrated approach demonstrates that the HK model parameters (ρ, δ, μ, γ) can be directly associated with social and digital law. Participation and transparency mechanisms support Q's sustainability by increasing ρ; the destructive effects of δ, μ, and γ on the system are limited.

XII. CONCLUSION AND FUTURE PERSPECTIVE: THE 2035–2065 PERIOD OF MODERN CIVILISATION WITHIN THE HK MODEL AND SYSTEMIC PRUNING FRAMEWORK

This study has addressed, with mathematical and logical integrity, the transformations that modern civilisation will confront during the 2035–2065 period within the framework of social entropy, the HK model, and the concept of systemic pruning.

1. Empirical Calibration and Parametric Integration

Q and N variables: indicators measured by global institutions have been directly integrated into the HK model:

- **Q (institutional/qualitative capacity):** proxy indicators such as WIPO GII, PISA scores, academic publication, and patent production.
- **N (system pressure/quantitative burden):** World Bank WGI data, IDC DataSphere data volume, and energy consumption metrics.

This calibration enables the numerical simulation of Q and N as time series in the ODE system and permits the empirical monitoring of bifurcation points.

2. Platform Economy and Algorithmic Entropy Integration

Data-energy analogy: platforms are expanding global pressure by increasing N with the digital component N_dig: N_total = N_physical + λ·N_dig. Scenario E (algorithmic lock-in): assuming P(t) → 1 and γ = 0.12, it has been numerically demonstrated that the 2035 rupture is advanced by approximately 25 time steps.

3. Stochastic Extension and Uncertainty Bands

Wiener process integration:

$$dQ = [\rho \cdot Q(1-Q) - \delta \cdot Q - \mu \cdot S \cdot Q]dt + \sigma_Q \cdot dW_Q$$

Logical conclusion: probability bands in the vicinity of the bifurcation point quantitatively represent the uncertainty of Φ's approach to the critical threshold and the stochastic nature of systemic risks.

4. Policy Action Plan and Operationalisation of Scenario D

HK parameters and policy instruments:

- **δ reduction** → minimisation of pressure and coordination deficiency
- **ρ augmentation** → strengthening of self-repair capacity
- **β augmentation** → optimisation of entropy dissipation rate

- **α reduction** → ecological and resource-based entropy control

Mathematical implication: Scenario D, following the detection of the $\Phi > 1$ critical threshold, keeps the system's Q value above the critical lower threshold through coordinated interventions, minimising destructive pruning.

5. Historical Perspective and Humanity's 'Second Chance'

The examples of Rome, the Maya, the Soviet Union, and Britain demonstrate that systems which detect the $\Phi > 1$ condition early and apply coordinated interventions are able to restructure themselves following destructive pruning processes. The HK model provides the parametric and mathematical counterpart of this restructuring: high ρ and optimised $\delta/\mu/\gamma$ values guarantee civilisational sustainability by preventing Q from falling below the critical lower threshold.

6. Academic and Policy-Oriented Implications

15. Theoretical clarity: the interaction among social, ecological, and digital parameters within the HK model has been clearly defined.
16. Mathematical consistency: deterministic and stochastic ODE/SDE systems have been combined with critical threshold and bifurcation analysis.
17. Political logical integrity: the Scenario D vs. Scenario E comparison quantitatively demonstrates the timing and mechanism of policy interventions.
18. Strengthening of academic standards: literature references (Tainter, Holling, Zuboff, Raworth, Floridi, Habermas) support the HK model's theoretical and practical power.

7. Future Perspective (Post-2065)

- **Focus:** qualitative deepening rather than quantitative growth; the preservation and augmentation of Q.
- **Core mechanisms:** augmentation of ρ through participation and transparency; reduction of γ through control over algorithmic entropy and platform monopoly; reduction of α through ecological and resource management.
- **Conclusion:** humanity's 'Second Chance' depends on the HK parameters shaped by today's decisions, institutional capacity, and digital law. The management of the critical threshold between 2035 and 2065 can be defined as the decisive period for the construction of a new civilisation.

Figure 9 — Post-2065 Civilisational Stabilisation Trajectory

Simulated $\Phi(t)$ under Scenario D with full coordinated intervention package (2035–2095). Markers: pruning onset, intervention package activation, post-crisis stabilisation, 2065 target threshold ($\Phi = 0.75$). [Figure generated from Python/Monte Carlo output.]

Figure 9. Post-2065 Civilisational Stabilisation — HK model projected recovery trajectory under Scenario D.

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